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A glimpse of dual nature of complex number

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Abstract

Ever since the inception, complex numbers remained without apt mathematical value i.e. they have not been nominated on a position on the number line. This document offers a hypothesis that can give complex numbers a genuine (mathematical) value. The study scrutinizes how a complex number performs in terms of real numbers thus finding a way to give complex numbers a real value. Al though the hypothesis may seem unacceptable its mathematical and physical implication, conferred in the paper, are unkind of such an answer. The very fact that a complex number can be allotted a real value can prove to be practical especially in the field of complex analysis.

Keywords: complex number, imaginary value, real value, square root, duality, number line

1. Introduction

A complex number is a number which can be put in the form a+*i*b, where a and b are real numbers and *i* is called the imaginary entity. Mathematically *i*= √−1. Complex numbers are inert imaginary numbers, regardless of the huge progression in mathematics. Many mathematicians over the years have strived to give a quantitative value to complex numbers but all of them failed to give ‗i‘ a comprehensive value. Complex numbers be compared to the opponent that negative numbers don't have any real value which means they cannot be positioned on the number line. For e.g. the 'pi' which has a value 3.14... and the decimals go on. Though pi's decimals aren't definite, still there is a precise idea in relation to its placement on the number line. But if a complex number 3+*i*5, there is utterly no way to have any idea of where that number would be on the number line. In this paper, the attempt is given to provide a solution for the real value nature of complex numbers. During the research work, the way are found by which real values to a complex number can be assigned, which technically has no distinct value. By doing so, it surprisingly conveys the possession of dual nature of mathematics. Duality is a well-known concept in physics wave matter duality etc. But such a concept has certainly not subsisted in mathematics. According to mathematics, an entity can have only one value or one value among two or more possible values. Mathematics doesn't authorize any variable to "hold" more than one value. This hypothesis put forward that complex numbers have a dual value nature in terms of real numbers, consequently the title Dual Real Value Nature of Complex Numbers. The fact that complex numbers can be articulated in terms of real values could confirm to be precious for prospective developments in mathematics.

The hypothesis for the value of *‘i’*

By mathematical definition, i2 = - 1*1

- i*i = -1*1

This can be further classified as i = +1 AND i = -1

It‘s the only way to compare the expression and it entails that ‗ i‘takes both values of -1 and +1. This does indeed contradict the fundamentals of mathematics.

Thus, a complex number 3+ i 5, it would have both real values of -2 & 8. This show how a complex number behaves in terms of real numbers.

Mathematics has no specification for such a activities. Mathematics stands by a principle of one variable being capable of seizing only one value. Even in the case of conventional square roots, a variable can have only one of the two values, either the +ve answer or the -ve answer. Likewise, computers also work only on this principle. A single memory box can‘t hold more than one value.

This duality principle anticipated at this time that negative numbers received when they were introduced. At that time, the concept of a number being negative was simply not imaginable. But nowadays negative numbers are as significant as the positive ones, in not only mathematics but also in physics, chemistry and all type of sciences. Mathematical, duality that is proposed, also is the same case. Though it seems unacceptable for an entity to have more than one value, it is the only way to answer a question that even the most modern mathematics principles can‘t answer i.e. giving complex numbers a ―plausible‖ value. This hypothesis totally betrays mathematics because now a variable can hold two values (if that variable is assigned to a complex number). Here, the attempt is made to prove this hypothesis. But to illustrate the logical validity of hypothesis, some testimony that explain that the above counter does subsist.

Analysis of square roots

The square root of a positive number is well known and well defined.

Example 1

√36 = ± 6; In words, Square root of 36 is +6 OR – 6.

But using the outcome conferred in the paper, the square root of a negative number can be found.

Example 2

√ -16= ±4; In words, Square root of -16 is + 4 AND – 4.

The symbol is just to bring to light AND relation of the roots while ± emphasizes OR relation amongst the roots.
Square root of a positive number presents a + OR – answer. Whereas, Square root of a negative number furnishes a + AND – answer.

Mathematically,

$x = \sqrt{-36} = (6) \wedge (-6)$; $x = \sqrt{36} = (-6) \vee (-6)$

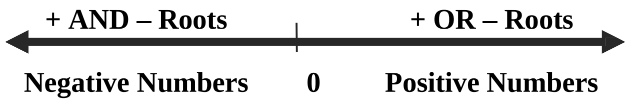

Fig 1: Symmetry of Square Roots on the Number Line

While analyzing the number line, the symmetry of arrangement can be visualized. The disparity in the response of the two square roots is fundamentally on the basis of the relationship between the roots – Conjunction or Disjunction. However the AND-OR relation might not have any straight connotation from Boolean algebra or set language, if such implications exists, then that would be the evidence to hypothesis.
This arrangement is a demonstration to the hypothesis but on further research, this result has direct consequences in physics and its existence can be mathematically revealed as assumed before.

Mathematical proof of existence
Cauchy-Riemann Equations: The differential equations give the necessary condition for a complex function f(z) to be regular.

If w = f(z), where w = u + i v and z = x + i y and since u and v are both functions of x and y and therefore we can write
w = f(z)= u (x,y) + i v (x,y)

Now if w is differentiable at a given point z, the limiting value must tend to a certain finite limit as $\Delta z \rightarrow 0$ from any direction.

$$\Delta z = \Delta x + i\,\Delta y$$

If Δz is wholly real then $\Delta y = 0$, differentiating w with respect to x

$$\frac{dw}{dx} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

Similarly again if Δz is taken wholly imaginary then $\Delta x = 0$ and we get the limiting value again by differentiating w with respect to y.

Output will be: $\frac{dw}{dx} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$

Till the calculation of the two limiting values, the derivation is same. The subsequent steps are
a) Persists as per the conventional descent
b) Uses the hypothesis considered in the paper

A) Regular Derivation
Since the function is differentiable, the two limiting values so obtained must be identical

$$\frac{dw}{dx} \equiv \frac{dw}{dy}$$

Equating real and imaginary parts the Cauchy-Riemann Equations are obtained

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \;\&\; \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

B) Hypothesis Based Derivation
The following derivation is extremely tedious compared to the conventional derivation. But it can be argued that the hypothesis is correct if I can arrive at the Cauchy-Riemann equations using my hypothesis in the derivation.

In $\frac{dw}{dx} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ and $\frac{dw}{dy} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$ taking the ‗i‘ and replacing it with ⊕1 as posed in the hypothesis

$$\frac{dw}{dx} = \frac{\partial u}{\partial x} \oplus \frac{\partial v}{\partial x} \;\&\; \frac{dw}{dy} = \frac{\partial v}{\partial y} \oplus \frac{\partial u}{\partial y}$$

Since it‘s a ⊕ sign before $\frac{\partial u}{\partial y}$ the ‗–‗sign becomes meaningless like that in ±.

Now equating the two limiting $\left(\frac{dw}{dx} \equiv \frac{dw}{dy}\right)$ taking into account the AND relation four equations can be obtained.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} + \frac{\partial u}{\partial y} \quad (1); \qquad \frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} + \frac{\partial u}{\partial y} \quad (2);$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \quad (3); \qquad \frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \quad (4);$$

Adding (1) & (2) | Adding (1) & (3)

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} + \frac{\partial u}{\partial y} \quad (5); \qquad \frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} \quad (6);$$

Adding (1) & (4) | Adding (2) & (3)

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad (7); \qquad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad (8);$$

Adding (2) & (4) | Adding (3) & (4)

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} \quad (9); \qquad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \quad (10);$$

Subtraction is not done since it engross a change in sign of one of the equations alone, disrupting the vital relationship between the equations because each equation isn‘t a separate entity but only based on the dual nature (AND principle). It can be pragmatic equations (1), (2), (3), (4) looks identical but for the signs in between them.
Equations (7) and (8) are the same (again proving AND relation between the subsequent equations as well, though the equations look different they are actually the same) and are one of the Cauchy-Riemann equations.
Equating (5) & (6)

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial y} = \frac{\partial v}{\partial y} - \frac{\partial v}{\partial x} \quad \Rightarrow \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad (12);$$

Similarly equating (9) & (10)

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \quad => \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad (13);$$

Equations (12) and (13) are the same and are the other Cauchy-Riemann equation.

As a result both the obligatory differential equations are attained, but on differently equating (5) & (10) using the converse of the hypothesis (replacing the corresponding + AND – real equations into a single complex equation with the imaginary part _i'

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} + i\frac{\partial u}{\partial y}$$

Similar equating of (6) & (9)

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

The mathematical significance of these two equations is that if those two equations are equated again and simplified further, we get the second Cauchy-Riemann equation

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

This is like a Round-Robin process where auxiliary combinations of equations and their subsequent simplifications. The results only end up with the two basic differential equations. Though this method is a dreary means to obtain the Cauchy- Riemann equations, it confirms mathematically, the existence of AND relation in complex numbers, functions and variables, as the two basic differential equations are undeniably achieved.

Physical verification of existence

Consider a straight wire carrying steady current along z-axis. The magnetic field is along the plane (x-y plane) perpendicular to the wire (along z-axis). The electromagnetic field is a complex number. Since the electric field is fixed and unidirectional through the wire, it becomes the real part of the complex number - electromagnetic field. So naturally the magnetic field becomes the imaginary part of the complex number.

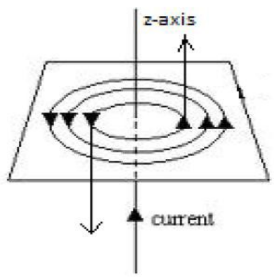

Fig. 2 Magnetic field due to a Straight wire carrying field

As represented in figure 2, the two lines are drawn on the plane of the magnetic field and they point towards the direction of the magnetic field at the two respective points. The magnitude of the field is the same but they are opposite to each other in direction at those two points. Since magnetic field is a vector it simply means that one is the negative of the other.

This shows that at the given moment of time, the field exists in such a way and in such a direction at the two points, indicating the + AND – relation of the value of i (magnetic field is considered as the imaginary part).

This property is not restricted to the two points in consideration but for any point in a magnetic field due to a straight wire carrying current there will always be another point where the magnitude of the field is same but the direction of the field is exactly opposite to the direction of field at the former point. This is a direct consequence of the hypothesis developed

Now why is the magnetic field circular and not just along those two converse directions? It's a matter of possibility. Let's presume the field to be circular.

There can be _p' number of orientations feasible along the perimeter of the circle for those two diametrically opposite field lines. Now the probability that those two field lines will ex ist in one of _p' number of orientations is 1/ p.

$$\alpha = p * \varphi$$

_α' is probability of field being circular, _φ' is probability of two diametrically opposite field lines being present in any one of the orientations and _p' is total number of possible orientat ions which is actually a large number.

Probability of field being circular = p * (1/ p) = 1

This shows mathematically that the field has to be circular in nature. Concentric circles of magnetic field are generated due to the change in the magnitude of the magnetic field. It must be remembered that a complex number a + i b has a real part and a imaginary part. But the _b' in the imaginary part is a real number. So with respect to the above example _i'gives the magnetic field its circular nature while _b' determines the magnitude of magnetic field at a particular distance from the wire. Thus it can be shown mathematically why the magnetic field of a straight wire carrying current is concentric circles about the wire and this type of a field has been observed experimentally. The converse of this effect is observed in a solenoid carrying current generating a linear magnetic field.

Conclusions

This paper tends to justify a real value is consigned to _i'. Though the upshot is hard to accept, the symmetry of arrangement of square roots on the number line, the hypothesis based derivation which still leads to the same set of Cauchy-Riemann differential equations and the real life example – the electromagnetic phenomenon are proof to the occurrence of such an answer. The result gives mathematics an entirely new unparalleled approach. Now it's doable for an entity to hold more than one value at the same time which contradicts basic mathematics but gives it a whole new dimension and scope. By means of the result, _i' can finally be mathematically valued and that as said earlier would prove to be valuable in complex analysis and other fields of mathematics too.

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