International Journal of Advanced Scientific Research
ISSN: 2456-0421; Impact Factor: RJIF 5.32
Received: 21-05-2020; Accepted: 11-06-2020; Published: 22-06-2020
www.allscientificjournal.com
Volume 5; Issue 3; 2020; Page No. 09-15

Cubic spline interpolation & profile geometry effects on the potential flow solutions

Salah JS El-Ibrahim
Lecturer, College of Aviation Technology, P.O. Box, Hawally, Kuwait

Abstract
Many Computational Fluid Dynamics (CFD) codes used for the solution of the potential flow around two dimensional aerofoil sections normally use a cubic spline to represent the coordinate data of the given profile in order to provide smooth functions for the conformal mapping procedure (to map the profile into a circle and reflect the data onto a computational grid, with the need for the splined data to match the grid's resolution points on the profile's surface).
Depending on the supplied (input data) of the profile, the cubic spline may exhibit oscillatory behaviour that may reflect onto the solution of the flow parameters and make it unreliable.
The present work attempts to test this behaviour with the Viscous Garabedian & Korn (VGK) code and to assess the code's performance with profiles having non-smooth geometries representing indentations (damages) to wings simulating accidental real-life damages.

Keywords: computational fluid dynamics; conformal mapping; cubic spline; modified NACA6410 aerofoils; numerical oscillations; viscous garabedian and korn (VGK) method

1. Introduction

In computational fluid dynamics (CFD) schemes designed for the two dimensional (2-D) potential flow calculations around an immersed body, the coordinate data of the body (the profile shape) is usually needed for the purpose of the conformal mapping procedure used to transform the profile contour to that of a circle [1] which presents itself as a profile where the potential flow parameters can be found on its perimeter. The original profile's coordinate data needs to be supplied in a smooth function where the first and second derivatives exist along the profile and can be easily found at the supplied coordinates. To ensure this condition, the supplied data is normally represented by some form of a piece-wise spline (usually cubic). The splined data is supposed to be smoothly distributed and match the number of the grid points on the perimeter of the profile. In practice, however, the cubic spline exhibits unwanted oscillations which tend to create extraneous inflection points whether or not these exist in the original data points. These oscillations will contaminate the potential flow calculations particularly where supplied profile data do not represent that of a smooth curve (situations where damage to profile, indentations, ice accretion, excursions...etc. exist). The present work attempts to examine the effects of the cubic spline on the potential flow calculations about profiles with non-smooth region (s). The Viscous Garabedian & Korn (VGK) CFD code will be used as the study medium since it uses a cubic spline interpolation of the supplied profile coordinates, and since it has been shown previously [2] that the code is able to deal with aerofoil sections with indentations generated randomly or with polynomial functions. Computational experiments, using the VGK code, will be applied to modified NACA6410 aerofoil profiles. The profiles are modified by imposing certain indentations to the upper surface near the leading edge as will be defined below. Besides producing the potential flow parameters (surface pressures, surface velocities…etc.), the VGK code also produces the geometrical and the conformal mapping data: Input and produced (splined) aerofoil coordinates, first and second derivatives of the surface profile function, the mapping moduli, and the number of iterations with error residuals. The present study analyses these parameters to shed light on how close they simulate/represent the physical flow under question.

2. VGK and Geometry effects

Factors related to aerofoil and indent geometry that are expected to affect the numerical calculations include:

a. Smoothness of the profile-defining function
b. Global and local resolution of ordinate data (whole profile and within indentation respectively) and its relation with the computational (discretized) resolution
c. Local properties of the indentation geometry (length, depth and contour variation)

VGK places restrictive (though not impractical) requirements of smoothness and adequate data resolution within particularly rapidly changing profile gradients to ensure at least the existence and continuity of first and second order derivatives. Such requirements are needed to yield a smooth representation of the profile through the interpolating cubic spline function used in the conformal mapping procedure and in re-interpolating the supplied data to coincide with the discretized mesh of the computational domain. In practice the cubic spline exhibits unwanted undulations [3, 4] which tend to create extraneous inflection points whether or not these exist in the original data points. An extraneous inflection point of a cubic interpolant $s(x)$ in an interval $[x_i, x_{i+1}]$ is found to occur when the second central differences of the data points at x_i & x_{i+1} are of the same sign [3], i.e. when:

$$\alpha_i \alpha_{i+1} > 0 \quad (1)$$

where for the spline data point f_i and length of spline i[th] interval h_i

$$\alpha_i = \frac{f_{i+1} - f_i}{h_i} - \frac{f_i - f_{i-1}}{h_{i-1}} \quad (2)$$

This particular situation would seem very restrictive when applied to a non-smooth varying geometry representing a real-life damage (indentation), and the cubic spline interpolation in this regard would seem inefficient. This may give rise to spurious oscillations and unrealistic flow behaviour (non-physical separation regions with viscous calculations or the appearance of shock waves in the wrong locations). Examples of this are cited in the work of reference (3) where the use of exponential splines is suggested. However, this will not be of concern here as the exponential spline is not available within VGK. It is thus imperative that the efficiency of the cubic spline be examined in respect of its ability to represent a given profile with an indentation geometry that resembles a real-life situation.

3. Modified NACA6410 Aerofoils

3.1 RND10201.dat Profile

This is a NACA64010 profile modified by a randomly generated indent imposed on its upper surface within the region $0.1 \le \frac{x}{c} \le 0.2$ (where x/c is the chord-wise ordinate with respect to the chord (c)). Matlab [5] is used to generate the random data points. The generated data are normalized against the maximum value produced and multiplied by a certain number to represent the maximum depth of the indentation. The produced data are then subtracted from the corresponding y-ordinates (thickness ordinates) on the upper surface of the original profile. In this case the chosen maximum indentation depth was 0.007 with respect to the chord length. The indent geometry function is in fact the result of an initial cubic spline interpolant of the randomly generated data before imposing it onto the NACA64010 profile. This is done so as to provide the smooth continuous function (together with its first and second derivatives) required by VGK. Figure (1) shows the result of applying equation (1), where the positive regions are regions of expected spline-generated extraneous inflection points, ("expected" is used here because strictly speaking equation (1) should be applied to the spline data and not to the actual data). These are the regions expected to trigger geometry-related oscillatory behaviour in the numerical solution. The amplitude of these oscillations may be diminished through using a finer grid (the indentation generated data points need to match the grid points along the profile surface). Because this is a geometry-related problem, such oscillations will not be expected to diminish merely by increasing the number of iteration cycles in the potential flow field calculations as will be shown later in this section.

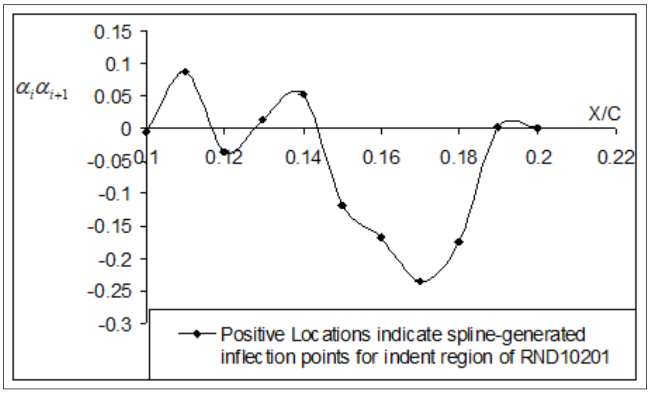

Fig 1: Application of equation (1) to the indent of RND10201.dat

The VGK-calculated mapping modulus provides an excellent check on the expected geometry-related behaviour; this is because the invariant (physical) velocity components are calculated from the mapped velocity components through the relation:

$$[u - jv]_{physical-domain} = \left[\frac{u - jv}{\frac{dF}{dz}} \right]_{Mapped-region}$$

with $\left(\frac{dF}{dz}\right)$ being the mapping function derivative. Hence any variations (or oscillations) in the numerical calculations of $\left(\frac{dF}{dz}\right)$ will be reflected in (u, v). In VGK the modulus of the mapping function is calculated for the mapped profile on the unit circle. This can be output by VGK and figure (2) shows the mapping modulus of the RND10201.dat profile where the existence of the oscillatory behaviour is evident and extends over a region beyond that of the indent. This automatically means that the same trend in oscillations will be present in the calculated velocity (and hence, pressure) distribution over the profile surface regardless of the free stream conditions, i.e. irrespective of whether or not hyperbolic regions exist in the flow.

The results of a viscous VGK run at M_∞=0.30 (well below the critical Mach number) and zero incidence are shown in figure (3). This verifies the fact that the existing numerical oscillations are geometry-related. Note also that such oscillations are not confined to the indent region but contaminate the whole of the upper surface.

The virtually identical results (not shown) are also obtained when the number of iterations is increased from (300) to (2500). Had the oscillations been due to the actual numerical solution of the full potential equation, then it would be expected that using 2500 iterations would produce some changes to the solution. The well-behaved convergence history also confirms that this is not so.

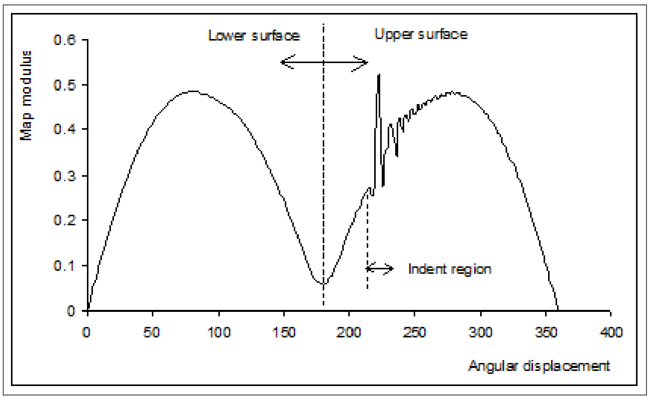

Fig 2: Mapping modulus for RND10201.dat

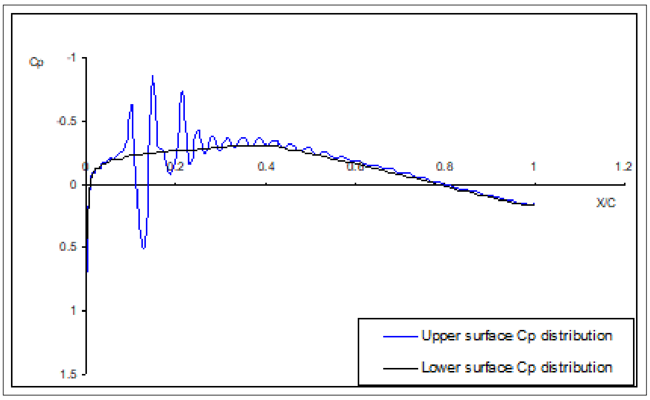

Fig 3: Pressure distribution for RND10201.dat at M_∞ =0.3 (Results after 300 and 2500 (not shown) iterations match exactly)

3.2 Profiles with indents near the leading edge

The shortest length of any indent in a profile is given by the smallest difference between two consecutive x-ordinates and for analytically generated aerofoils only practical considerations limit this choice. For the purpose of the present work the NACA 64010 data is generated by the "Airfols" code [6] with a resolution (δx) distribution as follows:

$$0 \le \frac{x}{c} \le 0.01 \quad \delta x = 0.00025$$

$$0.01 < \frac{x}{c} \le 0.1 \quad \delta x = 0.0025$$

$$0.1 < \frac{x}{c} \le 1 \quad \delta x = 0.01$$

Accordingly, the shortest length is given by 0.00025, which represents a length increment of 0.25mm for a 1.0-meter chord over the first 1.0% of the chord from the leading edge. Since

this region of the wing is usually the most susceptible to accidental damage, and since this is also the most aerodynamically active (high flow gradients), then it will be of practical use to consider the VGK performance with indents in this region. A number of indent profiles are suggested for this purpose, these are summarised in table (1). All the profiles are generated through a sixth power polynomial with the maximum depth occurring in the middle of the indent length. All VGK runs are performed at M_∞ =0.40, zero angle of incidence, R_e =20x10^6 with transition points chosen at x/c=0.03 for both surfaces, and with VGK parameters [7] set at their default values.

The above profiles were generated on the basis of keeping the indent length constant (at 0.0025) while gradually increasing the maximum depth until VGK calculations diverge then, with this maximum depth kept constant, the length is gradually increased until the VGK calculations are again convergent. For the first two profiles of table (1), i.e.: I0002685.dat and I0009685.dat, VGK produced no significant changes in the pressure distribution. For the I0009685.dat profile there is a noticeable discrepancy between the upper and lower surface pressure distributions starting to appear at about x/c=0.004 (well before the start of the indent at 0.006). This is also slightly apparent for the I0002685.dat profile and can be seen as a cross-over of the pressure and suction sides at about x/c=0.00725; the point of mid-length of the indent (maximum indentation). Mapping calculations converged after 11 and 14 iterations (total default number of iterations in VGK =30) for the shallower and deeper indents respectively, and potential calculations converged within a residual of (4x10^{-6} approx.) after 300 iterations (default number in VGK) for both profiles.

With the I0029685.dat profile the VGK results are characterised by strong oscillations that contaminated the whole solution domain but with diminishing amplitude away from the indent. This is shown in figure (4) where it is also apparent that the discrepancy in the pressure distribution between the upper and lower surfaces away from the indent is more pronounced. Although VGK seems to converge quite well in this case (residual being at 6x10^{-6} after 300 iterations), the produced results are not necessarily representative of a totally reliable solution. This can be judged by examining the VGK-mapping results. Two things are looked at in this regard; the number of mapping iterations necessary to reach the required unit circle in the computational plane, and comparison between the actual (supplied) profile and the produced one (i.e. the cubic spline-fitted one on which VGK performs the discretized calculations). In VGK the maximum number of mapping iterations is 30 and a history of the maximum error in the curvilinear profile perimeter s is given after each iteration cycle. If this number is totally exhausted then this is an indication of a questionable mapped profile, that is the error in (s), is not diminishing or it needs more iterations. The number of iterations can be increased manually in VGK control file (called VGKCON), however it has been found by the present author that in all cases where this was done the error in s did not diminish and could even be amplified.

A comparison between the supplied I0029685.dat profile and the VGK representation is shown in figure (5). Here it is apparent that the profile is slightly distorted by an overall reduced upper surface thickness. Had the supplied data been less resolved within the indent region-see inset of figure (5)- then this error may have been reversed (a thicker profile may have been produced). Also notice the distorted representation of the indent region itself (inset). This is obviously due the large difference in resolution between the supplied and the produced data- (within this region, 13 supplied data points are represented by a mere 3 and two of these are placed outside the original indent region). This also explains the earlier (upstream) predicted location of the represented indent region.

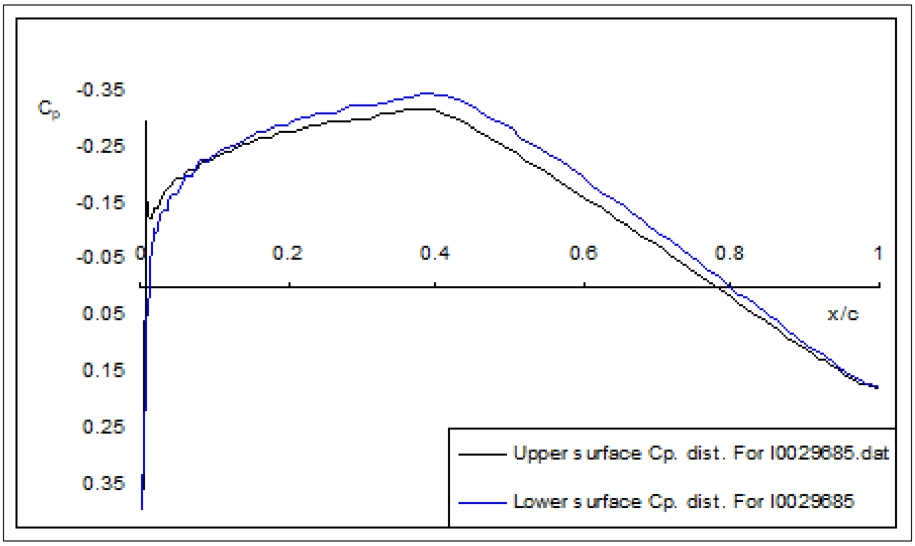

Fig 4: Pressure Coefficient (Cp) distribution for I0029685.dat

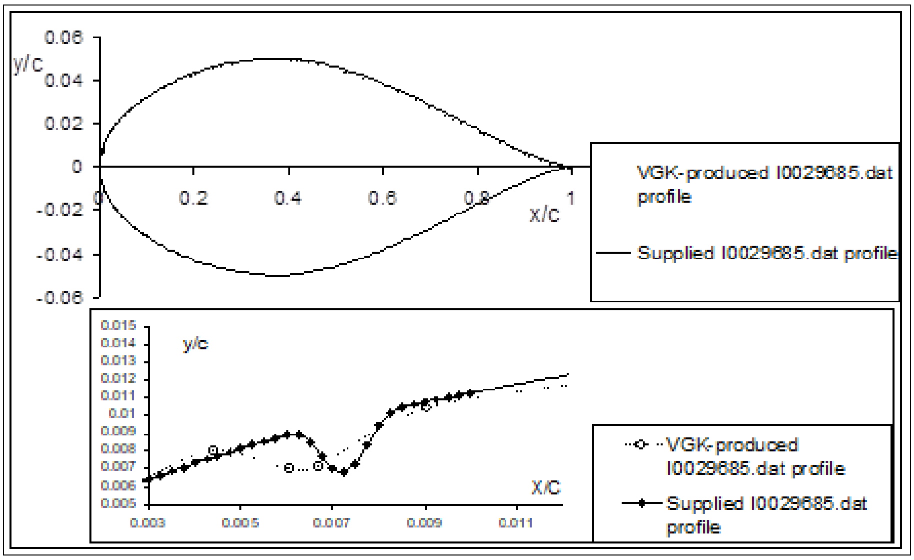

Fig 5: Supplied and VGK-produced I0029685.dat profile

The VGK run for the I0030685.dat diverged after only 105 potential-field iterations, but surprisingly convergence was achieved for the I0035685.dat profile that has a deeper indent over the same length. Such convergence, however, is misleading as the produced profile is grossly misrepresented particularly within the indent region. Not only the indent region in both profiles is greatly distorted but also the produced data resolution is in error. Note the difference between the two profiles regarding this data resolution-. These errors in both the produced profile and data resolution are due to the error in the final computed value of the profile chord, c, at the end of the mapping iteration cycles as final re-scaling of curvature (and ordinates) is normalised with the new chord. The source of error in c comes from updating the profile curvature during the mapping iteration so that the mapped chord is defined by the mapping procedure [8]. To see this, the maximum number of mapping iterations is reduced to 15 instead of 30, thus changing the final value of c. This indeed produced a converged (though still not representative) solution for the I0030685.dat profile. Moving to the next set of profiles, table (1), where the depth is kept constant while gradually increasing the length, similar trends to these experienced with the first set are obtained. VGK runs converged for all of these profiles except the longest (i.e. I0036175.dat). This seems surprising at first sight, since the longer the profile the smoother the profile's curvature and hence fewer problems should be expected with VGK. The reason however, is attributed to the mixed resolution of the supplied data over this length where the mid-point of the indent in this case is at x/c=0.01175, so when the indent profile inflects (changes direction after passing the maximum indentation point) the resolution is reduced from $\delta x = 0.00025$ to $\delta x = 0.0025$. In fact, the rest of the profiles exhibit a similar resolution trend except for I0036100.dat, as can be seen in figure (6). All in all, a combination of a short length indentation and/or a mixed resolution gives rise to the VGK results being unreliable even when they seem to converge; giving solutions similar in trend to that shown in figure (4) above. The situation is worsened when the discretized data points (VGK-produced resolution) are far less than the supplied ones in number. Here fewer data points represent the same length and accordingly, very small variations in profile may not be picked up correctly or may not even be represented at all. This situation may be remedied by using a finer grid, and/or by increasing the resolution in the region of interest so that supplied and discretized data points are closely comparable. The effect of increasing the number of data points is demonstrated for the I0036175.dat by re-defining the data within the indent through a cubic spline (using Matlab) so that the data are evenly distributed throughout the indent region. With the new data resolution VGK converges to a solution but still misrepresents the original profile (not shown here).

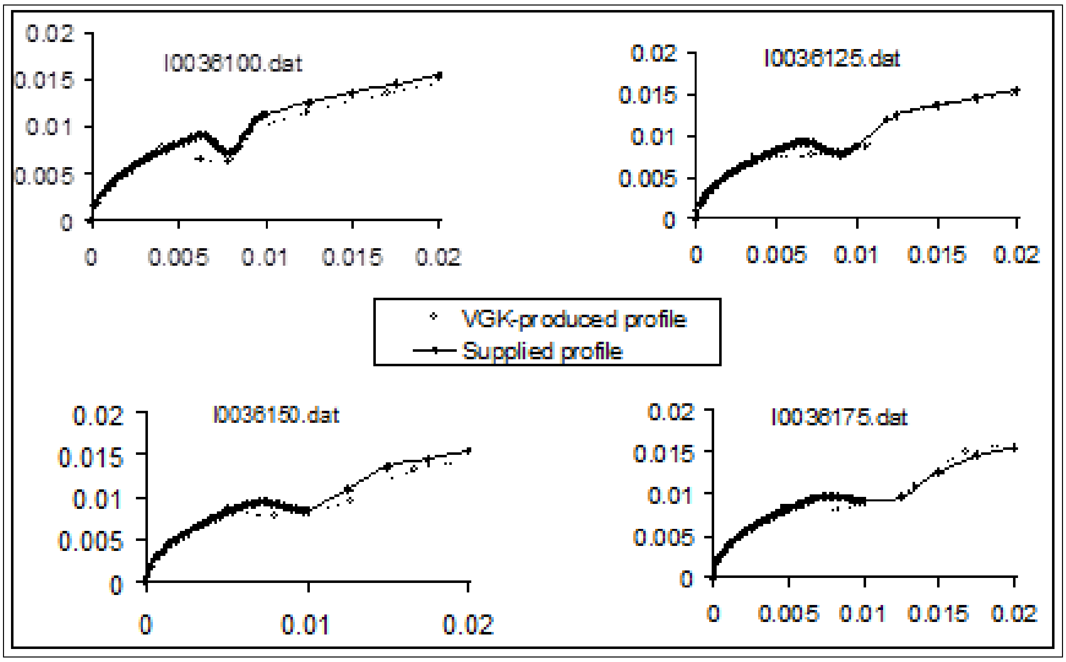

Fig 6: Comparison between supplied and VGK-produced data (same depth different length profiles)

More indented profiles (not shown, for brevity) with different depths and lengths, but all within $0.005 \le \frac{x}{c} \le 0.0375$ were also tried with VGK. The same trends as experienced above were also prevalent with convergence only achieved for depths not exceeding 0.006 for most cases.

4. Discussion of results

The preceding discussion shows that VGK is sensitive to curvature changes particularly in a region very close to the leading edge. Assessing the results of a situation where an indent exists near the leading edge requires careful considerations of the mapping results and the possible errors these can cause to the whole flow solution. Nothing has been said so far to relate the VGK performance to the length, size and position as the parameters characterising the indent geometry. For the indent profiles suggested above, it is not an easy task to find a relationship between the convergence, say, of VGK and the geometrical properties of the indent represented by a certain function of these parameters. This is due to the fact that other than purely geometrical considerations play a decisive role in both the convergence and accuracy of the VGK results, e.g. data resolution variations within and around the indent. In order to arrive at a reasonable geometrical parameter that could be applied in a general sense, it is important that this parameter shall relate the depth, the chord-wise length and a contour derivative-related function so that the position of the indent is also accounted for. In this regard the following formula is suggested by the present author, to characterise the size and position of the indent:

$$A\Re = \left(1 - \frac{y_j^I}{y_j^O}\right)\left(\frac{y_{j+1}^I - y_j^I}{y_{j+1}^O - y_j^O}\right)\frac{\Delta r}{\Delta \theta} \qquad (3)$$

where y refers to the normal to chord ordinate of the profile, with subscripts referring to the data point in question and the (I,O) superscripts referring to the indented and original profiles respectively. The first bracket in equation (3) relates the local depth of the indent to the local original depth of the profile (half-local original thickness); the second bracket relates the local first derivatives of the indented and original profiles. The inclusion of the $\frac{\Delta r}{\Delta \theta}$ term indicates the possible dependence of the convergence process on the grid resolution with the depth of an indent being a certain multiple of Δr and the length a certain multiple of $\Delta \theta$. Upon applying equation (3) individually to all the generated indented profiles (related to the original NACA64010 aerofoil), and with $|A\Re|$ being the maximum absolute value of $A\Re$, it is generally found that when:

a. $|A\Re| < 0.5$

VGK convergence is normally guaranteed with acceptable reliability (conformal mapping usually done in less than 30 iterations unless data resolution problems prevail).

b. $0.5 < |A\Re| < 1.0$;

VGK convergence is guaranteed but reliability of solution is questionable.

c. $1.0 < |A\Re| < 2.0$;

d. VGK convergence is not guaranteed with divergence almost certain when $|A\Re| < 2.0$, even if convergence takes place the solution is highly unreliable.

The sequence (a-c) above is compatible with an increasing depth and a decreasing length of the indent coupled to an increasing variation in the first derivative. It must be emphasised here that equation (3) and the arguments above were only applied to the NACA64010 aerofoil and all indentation geometries imposed on it. However, by virtue of the ratios used in the $|A\Re|$ parameter a generalisation of the above to different aerofoil sections could easily be applied and tested.

The shortest indent length possible with the NACA64010 aerofoil within the range $0.10 \le \frac{x}{c} \le 1.0$ is 0.02 with the provided resolution. More indented aerofoils are also generated where the indents occur within the region $0.1 \le \frac{x}{c} \le 0.12$, having depths of 0.0009,0.003,0.009,0.01, with one indent extending from to x/c=0.14 having a depth of 0.01. These profiles and their designations are summarised in table (2) below. Again, VGK produced converged solutions in accordance with the predictions of equation (3). For the first two of these, the solution converges without exhausting the total number of mapping iterations (only 13 and 15 mapping iterations performed in each case respectively). Numerical oscillations in the solutions of VGK are still evident for all of these profiles with increasing amplitude and chord-wise extension as the depth is increased until divergence is encountered with a depth of 0.01. With increasing $|A\Re|$ the tendency to produce unreliable solutions (marked by increasing amplitudes of numerical oscillations) is approached. This is obvious in figure (7) comparing the pressure distributions for the second and third profiles ($|A\Re| = 0.25\,\&\,1.78$, respectively).

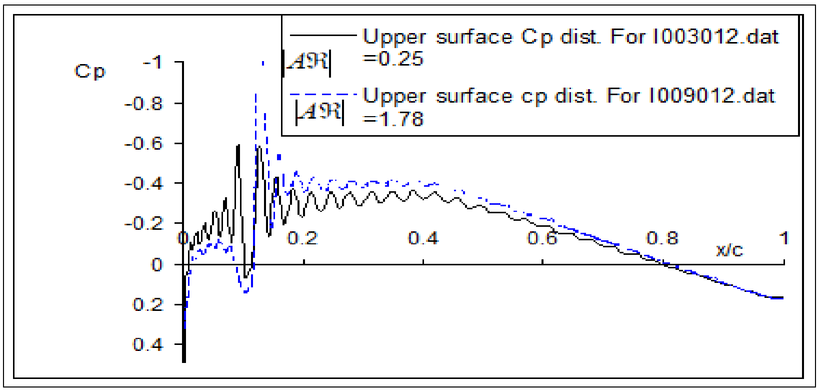

Fig 7: Increasing $|A\Re|$ increases tendency towards unreliable solutions

5. Conclusions

The cubic spline used in the VGK method was shown to exhibit oscillatory behaviour in representing some aerofoil profiles when the input data do not follow a smooth function or where adverse changes in curvature take place between consecutive points (e.g. non-smooth indentations) specially near the leading edge of the profile. Aerofoil indent geometry effects were shown to be effects due to curvature changes, ordinate data resolution, size and location of the indentation in relation to the original profile. These factors combined with the grid resolution effects prompted the use of the $A\Re$ parameter as stipulated by equation (3) and suggested by the present author. Guidelines on the value of $A\Re$ and its effects on the overall calculations of the VGK method were provided and showed that as the value of $|A\Re|$ increases (above 0.5), the VGK results tend to be unreliable.

6. Recommendations

When dealing with non-smooth changes to the curvature of a profile, the reliability of a CFD solution by must be checked by considering the profile-data produced according to the splining procedure, the number of iterations utilised and whether convergence is achieved before exhausting the default number of iteration cycles used in the code. It is also important to consider the number of data points produced in relation to the computational grid resolution to check whether the data points are truly represented in number and original position on the profile.

If the used software provides an output data of the profile's curvature and mapping moduli, then it is recommended to investigate the data graphically to check for any numerical oscillations. The shape parameter $|A\Re|$ can also be calculated and utilised as a guideline on the reliability of the solution as discussed in this work.

Future developments of the VGK code (or any similar CFD codes) may utilise an exponential spline interpolant of the given profile data in the hope of avoiding the oscillatory behaviour of the cubic splines.

Table 1: Suggested indent profiles for investigating VGK performance

Profile designation	Indent extent along chord	Indent length w.r.t chord	Indent depth w.r.t chord
I0002685.dat	$0.006 \le x/c \le 0.0085$	0.0025	0.0002
I0009685.dat	$0.006 \le x/c \le 0.0085$	0.0025	0.0009
I0029685.dat	$0.006 \le x/c \le 0.0085$	0.0025	0.0029
I0030685.dat	$0.006 \le x/c \le 0.0085$	0.0025	0.0030
I0035685.dat	$0.006 \le x/c \le 0.0085$	0.0025	0.0035
I0036100.dat	$0.006 \le x/c \le 0.0100$	0.004	0.003
I0036125.dat	$0.006 \le x/c \le 0.0125$	0.0065	0.003
I0036150.dat	$0.006 \le x/c \le 0.0150$	0.009	0.003
I0036175.dat	$0.006 \le x/c \le 0.0175$	0.0115	0.003

Table 2: More indent profiles for investigating VGK performance

Profile designation	Indent extent along chord/indent length/indent depth	Maximum $\lvert A\Re \rvert$	Mapping iterations
I0009012.dat	$0.10 \le x/c \le 0.12$ /0.02/0.0009	0.038	13
I003012.dat	$0.10 \le x/c \le 0.12$ /0.02/0.003	0.25	15
I009012.dat	$0.10 \le x/c \le 0.12$ /0.02/0.009	1.78	30
I01012.dat	$0.10 \le x/c \le 0.12$ /0.02/0.01	2.17	30
I01014.dat	$0.10 \le x/c \le 0.14$ /0.04/0.01	1.37	30

7. References

1. Sells CCL. "Plane subcritical flow past aerofoils". Proc. Roy. Soc. A, 1968; 308:377-401.
2. Salah JS EL-Ibrahim. “Prediction of the effects of aerofoil surface irregularities at high subsonic speeds using the viscous Garabedian & Korn method”. PhD thesis. Department of Civil Aeronautical and Mechanical Engineering, University of Hertfordshire, U.K., unpublished work, 2000.
3. McCartin BJ. "Applications of exponential splines in computational fluid dynamics". AIAA Journal, 1983, 21(8).
4. Belmari T, Bakir F, Kouidri S. "Numerical stability conditions in the calculation of potential flow fields". A technical note, AIAA journal, 1999, 37(2).
5. Mathworks: Generating Random Numbers with Matlab: https://www.mathworks.com/help/matlab/ref/rand.html
6. Ladson CI, Brooks CW Jr, Hill AS, Sproles DW. "Computer Program to Obtain Ordinates for NACA Airfoils". NASA TM 4741, 1996.
7. Freestone MM. "VGK Method for two-dimensional aerofoil sections- part1: Principles and results". Draft Data Item, ESDU international, 1996.
8. Garabedian PR, Korn DG. "Analysis of transonic aerofoils". Communications on pure and applied mathematics, 1971; 24:841-85.