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On sequences of Diophantine 3-tuples generated through the pair (9,2) each with property D(-2), D(-9), D(-14), D(-17)

S Vidhyalakshmi[1], T Mahalakshmi[2], MA Gopalan[3]

[1, 2] Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy, Tamil Nadu, India

[3] Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy, Tamil Nadu, India

Abstract

This paper aims at formulating sequences of Diophantine 3-tuples based on the Diophantine 2-tuple with properties $D(-2), D(-9), D(-14), D(-17)$ respectively.

Keywords: Diophantine 3-tuple, sequence of Diophantine 3-tuples

Introduction

The problem of constructing the sets with property that product of any two of its distinct elements is one less than a square has a very long history and such sets have been studied by Diophantus. A set of m distinct positive integers $\{a_1, a_2, a_3, \ldots, a_m\}$ is said to have the property $D(n), n \in Z - \{0\}$ if $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$ or $1 \le j < i \le m$ and such a set is called a Diophantine m-tuple with property $D(n)$.

Many Mathematicians considered the construction of different formulations of diophantine triples with the property $D(n)$ for any arbitrary integer n [1] and also, for any linear polynomials in n. In this context, one may refer [2, 12] for an extensive review of various problems on diophantine triples.

This paper concerns with the construction of sequences of diophantine 3-tuples (a, b, c) such that the product of any two elements of the set added by (-2),(-9),(-14),(-17) in turn is a perfect square.

Sequence: 1

Let $\mathbf{a = 9}$, $\mathbf{b = 2}$

It is observed that

$ab - 2 = 16$, a perfect square

Therefore, the pair (a, b) represents diophantine 2-tuple with the property $D(-2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 - 2 = p^2 \tag{1}$$

$$bc_1 - 2 = q^2 \tag{2}$$

Eliminating c_1 between (1) and (2), we have

$$bp^2 - aq^2 = (b - a)(-2) \tag{3}$$

Introducing the linear transformations

$$p = X + aT, \quad q = X + bT \tag{4}$$

in (3) and simplifying, we get

$$X^2 = abT^2 - 2$$

which is satisfied by $T=1$, $X=4$
In view of (4) and (1), it is seen that

$$c_1 = 19$$

Note that (a,b,c_1) represents diophantine 3-tuple with property $D(-2)$
Taking (a,c_1) and employing the above procedure, it is seen that the triple (a,c_1,c_2) where
$c_2 = 54$
exhibits diophantine 3-tuple with property $D(-2)$
Taking (a,c_2) and employing the above procedure, it is seen that the triple (a,c_2,c_3) where
$c_3 = 107$
exhibits diophantine 3-tuple with property $D(-2)$
Taking (a,c_3) and employing the above procedure, it is seen that the triple (a,c_3,c_4) where
$c_4 = 178$
exhibits diophantine 3-tuple with property $D(-2)$
The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a,c_s,c_{s+1}) where

$$c_s = 9s^2 + 8s + 2,\ s = 1,2,3,\ldots$$

Sequence: 2
Let $a=9,\ \ b=2$
It is observed that
$ab-9=9$, a perfect square
Therefore, the pair (a,b) represents diophantine 2-tuple with the property $D(-9)$.
Let c_1 be any non-zero polynomial such that

$$ac_1 - 9 = p^2 \tag{5}$$

$$bc_1 - 9 = q^2 \tag{6}$$

Eliminating c_1 between (5) and (6), we have

$$bp^2 - aq^2 = (b-a)(-9) \tag{7}$$

Introducing the linear transformations

$$p = X + aT\ ,\ q = X + bT \tag{8}$$

in (7) and simplifying we get

$$X^2 = ab\,T^2 - 9$$

which is satisfied by $T=1$, $X=3$
In view of (8) and (5), it is seen that
$c_1 = 17$
Note that (a,b,c_1) represents diophantine 3-tuple with property $D(-9)$
Taking (a,c_1) and employing the above procedure, it is seen that the triple (a,c_1,c_2) where
$c_2 = 50$

exhibits diophantine 3-tuple with property $D(-9)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$c_3 = 101$

exhibits diophantine 3-tuple with property $D(-9)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$c_4 = 170$

exhibits diophantine 3-tuple with property $D(-9)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a, c_s, c_{s+1}) where

$$c_s = 9s^2 + 6s + 2 \ , \ s = 1, 2, 3, \ldots$$

Sequence: 3

Let $a = 9, \ b = 2$

It is observed that

$ab - 14 = 4$, a perfect square

Therefore, the pair (a, b) represents diophantine 2-tuple with the property $D(-14)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 - 14 = p^2 \tag{9}$$

$$bc_1 - 14 = q^2 \tag{10}$$

Eliminating c_1 between (9) and (10), we have

$$bp^2 - aq^2 = (b - a)(-14) \tag{11}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + bT \tag{12}$$

in (11) and simplifying we get

$$X^2 = abT^2 - 14$$

which is satisfied by $T = 1 \ , \ X = 2$

In view of (12) and (9), it is seen that

$c_1 = 15$

Note that (a, b, c_1) represents diophantine 3-tuple with property $D(-14)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$c_2 = 46$

exhibits diophantine 3-tuple with property $D(-14)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$c_4 = 162$

exhibits diophantine 3-tuple with property $D(-14)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$c_4 = 162$

exhibits diophantine 3-tuple with property $D(-14)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by

(a, c_s, c_{s+1}) where

$$c_s = 9s^2 + 4s + 2 \ , \ s = 1, 2, 3, ...$$

Sequence: 4

Let $a = 9, \ b = 2$

It is observed that

$ab - 17 = 1$, a perfect square

Therefore, the pair (a, b) represents diophantine 2-tuple with the property $D(-17)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 - 17 = p^2 \tag{13}$$

$$bc_1 - 17 = q^2 \tag{14}$$

Eliminating c_1 between (13) and (14), we have

$$bp^2 - aq^2 = (b - a)(-17) \tag{15}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + bT \tag{16}$$

in (15) and simplifying we get

$X^2 = ab\,T^2 - 17$

which is satisfied by $X^2 = ab\,T^2 - 17$

In view of (16) and (13), it is seen that

$c_1 = 13$

Note that (a, b, c_1) represents diophantine 3-tuple with property $D(-17)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$c_2 = 42$

exhibits diophantine 3-tuple with property $D(-17)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$\mathbf{c_3 = 89}$

exhibits diophantine 3-tuple with property $D(-17)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$c_4 = 154$

exhibits diophantine 3-tuple with property $D(-17)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a, c_s, c_{s+1}) where

$$c_s = 9s^2 + 2s + 2 \ , \ s = 1, 2, 3, ...$$

It is noted that, in each of the above sequences,the following relations are observed:

- The triple $(c_s, c_{s+1} + 9, c_{s+2})$ forms an arithmetic progression.
- $2c_{s+1} - c_{s+2} - c_s = 2(3s+4)^2 - (3s+7)^2 - (3s+1)^2$

In conclusion, one may attempt for obtaining sequences of higher order Diophantine tuples with suitable properties.

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