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Analysis queueing models with bulk arrival having removable server

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Abstract

In this paper it is assumed that the server is always available but there are many situations in which the server has to perform the secondary work other than to providing service to the customers which is in queue. In this paper we consider queueing models in which server are removed for an exponentially distributed random time when queue is empty.

Keywords: Poisson arrival, bulk arrival of customer, exponential distribution, queue discipline is FCFS, inverse Laplace transformation M/M/1 queuing model

Introduction

In this chapter, we consider that server is removed for an exponentially distributed random time when the queue becomes empty and made a policy in which server removed from service for secondary work. The idle time of the server may be utilized. In all these studies arrival and service intensities were taken homogenous. Esien and Tainter (1963) firstly worked with the assumption of the single arrival and service, have obtained steady state solutions of M/M/1 queue, where arrival and service intensities are subject to Poissonian jumps between two states. Many practical situations can be represented closely by this system. The rush hour problem can be taken after completion of a cricket match where all spectators/ customers leave the stadium can be considered as one state and during start of match when arrival of spectators/ customer can be consider ordinary periods as another state.

In this paper a queueing model with variable batch arrival where the server may be removed from the service facility for an exponential random time is consider.

The following assumptions describe the system:

1. Arrivals arrive in batches of variable size under Poisson, law with parameter λ.
   Prob. [there is j customers in an arriving batch] = a_j
2. The queue discipline is first come – First serve.
3. The service time distribution is exponential with parameterμ.
4. The various stochastic processes in the system are statistically independent.
5. The server will be removed from its service as soon as it becomes empty for a negative exponential distribution, with parameterθ.

Notation

$P_{i,j,R}(t)$ - Prob. that there are exactly i arrivals and j departures by time t and the server is in removed state
$P_{i,j,B}(t)$ - Prob. that there are exactly i arrivals and j departure by time t and server is busy.
P_{ij} - Prob. that there are exactly i and j departures by time'‘t';
$i > j > 0$.

Initial conditions

$P_{0,0,R}(0) = 1$
$P_{0,0,B}(0) = 0$

The difference – differential equations governing the system are:

$$P'_{i,i,R}(t) = -\lambda P_{iiR}(t) + \mu P_{i,i-1,B}(t)\left(1-\delta_{i,0}\right),\ i > 0 \qquad (1)$$

$$P'_{i,j,R}(t) = -(\lambda+\theta)P_{ijR}(t) + \lambda\sum_{m=1}^{i-j} a_m P_{i-m,jR}(t),\ i > j > 0 \qquad (2)$$

$$P'_{i,j,B}(t) = -(\lambda+\mu)P_{ijB}(t) + \lambda\sum_{m=1}^{i-j-1} a_m P_{i-m,j,B}(t),\left(i-\delta_{i-1,j}\right) + \mu P_{i,j-1,B}(t)\left(1-\delta_{j,0}\right) \qquad (3)$$

$$P_{i,j}(t)=P_{i,j,R}(t)+P_{i,j,B}(t)(1-\delta_{i,j})$$

$$P_{00R}(0)=1$$

$$P_{00B}(0)=0 \tag{4}$$

using Laplace transformation of eq. (1 – 4), we will get

$$S\overline{P}_{iiR}(s)-P_{iiR}(0)=-\lambda\overline{P}_{iiR}(s)+\mu\overline{P}_{i,i-1,B}(s)(1-\delta_{i,0})$$

$$(S+\lambda)\overline{P}_{iiR}(s)=1+\mu\overline{P}_{ii-1,B}(s)(1-\delta_{i,0}),\quad P_{iiR}(0)=1$$

$$i=0,\ j=0$$

$$\overline{P}_{iiR}(s)=\frac{1}{s+\lambda}$$

$$\overline{P}_{00R}(s)=\frac{1}{s+\lambda} \tag{5}$$

$$S\overline{P}_{ijR}(s)-P_{i,j,R}(0)=-(\lambda+\theta)\overline{P}_{i,j,R}(s)+\lambda\sum_{m=1}^{i-j}a_m\overline{P}_{i-m,j,R}(s)$$

$$(s+\lambda+\theta)\overline{P}_{i,j,R}(s)=\lambda\sum_{m=1}^{i-j}a_m\overline{P}_{i-m,j,R}(s)+P_{i,j,R}(0)$$

Taking $i=1,\quad j=0$, thent

$$\overline{P}_{i,0,R}(s)=\left(\prod_{m=1}^{i}a_m\right).\lambda^i\overline{\beta}_{i,j}^{\lambda(\lambda+\theta)}(s) \tag{6}$$

$$\overline{P}_{i,j,R}(s)=\left(\prod_{m=1}^{i}a_m\lambda^i\mu.\overline{\beta}_{1,i}^{\lambda(\lambda+\theta)}(s)\right)^{1-\delta_{i,j}}\left(\frac{\mu}{s+\lambda}\right)^{\delta_{i,j}}\overline{P}_{i,j-1,B}(s) \tag{7}$$

$$\overline{P}_{ijB}(s)=\frac{\lambda}{(s+\lambda+\mu)}\left(\sum_{m=1}^{i-j-1}a_m\overline{P}_{i-m,jB}(s)(1-\delta_{ij})+\mu\overline{P}_{i,j-1,B}(s)(1-\delta_{j,0})+\theta\overline{P}_{i,j,F}(s)\right) \tag{8}$$

Again using, i = 1, j = 0, then

$$\overline{P}_{i,j,B}(s)=\left[\left\{\left(\sum_{m=1}^{i}a_m\right)\left(\frac{\lambda}{s+\lambda+\mu}\right)^i\right\}\left\{\sum_{m=1}^{i}a_m\frac{\lambda^1\mu\theta\overline{\beta}_{i,1,j}^{(\lambda+\mu)(\lambda+\theta)}(s)}{(s+\lambda)(s+\lambda+\mu)(s+\lambda+\theta)}\right\}\right.$$

$$\left.\left\{\sum_{m=1}^{i}a_m\frac{\lambda^i\mu}{(s+\lambda+\mu)^{i+1}}\right\}\left(\frac{\mu}{s+\lambda+\mu}\right)\right]\overline{P}_{i,j-1,B}(s) \tag{9}$$

Using the Laplace inverse transformation of the above equation (5 - 9), which will give

$$P_{00R}(t)=e^{-\lambda t}$$

$$P_{i0R}(t)=\sum_{m=1}^{i} a_m.\lambda^i\beta_{1,i}^{\lambda(\lambda+\theta)}(t)$$

$$P_{i,j,R}(t)=\left(\left(\sum_{m=1}^{i} a_m\right)\lambda^i\mu\bar{\beta}_{i,j}^{\lambda(\lambda+\theta)}(t)\right)^{1-\delta_{i,j}}\left(\mu e^{-\lambda t}\right)^{\delta_{i,j}}P_{i,j-1,B}(t)$$

$$P_{i0B}(t)=\sum_{m=1}^{i} a_m\theta\lambda^i\left(\frac{e^{-(\lambda+\theta).t.i}t}{\lfloor i}\right)e^{-(\lambda+\mu)_t}$$

$$P_{i,j,B}(t)=\left\{\left(\sum_{m=1}^{i} a_m\right)\lambda^i\left(\frac{e^{-(\lambda+\mu)t}t^{i-1}}{\lfloor i-1}\right)^{(1-\delta_{i,i,j})}\prod_{m=1}^{i}a_m\lambda^i\mu\theta\beta_{1,1,i}^{(\lambda+\mu)(\lambda+\theta)}(t)\right\}$$

$$\left\{\sum_{m=1}^{i} a_m\lambda^i\mu\left(\frac{e^{-(\lambda+\mu)t}t^i}{\lfloor i}\right)^{1-\delta_{i,j-1}}.\left(\mu e^{-(\lambda+\mu)}t\right)^{\delta_{i,j-1}}\right\}P_{i+1,j-1,B}(t)$$

From equation (5 -9), it is clear

$$\sum_{i=0}^{\infty}\sum_{j=0}^{i}\left(\bar{P}_{i,j,R}(s)+\bar{P}_{i,j,B}(s)\right)=\frac{1}{s}$$

and

$$\sum_{i=0}^{\infty}\sum_{j=0}^{i}P_{i,j,R}(t)+P_{i,j,B}(t)=1$$

This will give the verification of our result.

Taking some of the cases

1. Exactly i units arrive in time are

$$\bar{P}_{i0}(s)=\sum_{i=0}^{i}\bar{P}_{ijR}(s)+\bar{P}_{ijB}(s)\left(1-\delta_{ij}\right)$$

$$\bar{P}_{i0}(s)=\left(\frac{1}{s+\lambda}\right)\left\{\left(\prod_{m=1}^{i}a_m\right)\left(\frac{\lambda}{s+\lambda}\right)^i\right\}$$

$$P_{i,0}(t)=\prod_{m=1}^{i}a_m\frac{(\lambda t)^i}{\lfloor i}e^{-\lambda t}$$

If there are no batch arrivals i.e. arrival's one by one
$\Rightarrow a_1 = 1, a_2 = 0, a_3 = 0$ ---- $a_m = 0$

$$\bar{P}_{i0}(s)=\frac{\lambda^i}{(s+\lambda)^{i+1}};i>0$$

taking laplace inverse we get,

$$P_{i0}(t)=\frac{(\lambda t)^i}{\lfloor i}.e^{-\lambda t};i>0$$

The total number of arrivals are not affected by removal period ‘θ’ of the server and the arrivals follow a poison distribution.

2. The mean number of arrivals in time t is

$$\sum_{i=0}^{\infty} i\overline{P}_{i0}(s)=\sum_{i=0}^{\infty} i\left\{\frac{\lambda^i}{(s+\lambda)^{i+1}}\right\};i^30$$

$$=\left\{\frac{\lambda}{s^2}\right\}$$

taking laplace inverse of $\sum_{i=0}^{\infty} i\overline{P}_{i0}(s)$ is

3. The laplace transform $\overline{P}_{0j}(s)$ of the probability $\overline{P}_{0j}(s)$ that exactly j customers have been served by time t we have

4. Numerical validity check of inversion of $P_{i,0}$ (t)

Prob. [Exactly i units arrive in time‘t’] = $P_{i,0}$ (t)

λ	a_m	t	I	$\prod_{m=1}^{i} a_m \frac{(\lambda t)^i e^{-\lambda t}}{\lfloor i}$	$P_{i,0}$ (t)
1	0.1	1	1	0.03678	0.03678
	0.2			0.07356	0.7356
	0.3			0.11034	0.11034
2	0.1	2	2	0.014648	0.014648
	0.2			0.02929	0.02929
	0.3			0.043944	0.04394

As batch size increases the Probability of Exactly i units arrive in time‘t’ is also increases.

Conclusion

we conclude from this that the total number of arrivals is not affected by removal period ‘θ’ of the server and the arrivals follow a poison distribution. Also, we come to result that increase in the batch size tends to increase in the arrival time of the customers.

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