



## Modeling call set-up success rate to Nigeria telecommunication using three-parameter nakagami-m distribution

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### Abstract

The research paper examined the quality of service of the Nigeria telecommunication service providers. The call set-up success rate was analyzed using three parameters Nakagami-m distribution. The parameter estimates of the Nakagami-m distribution was provided. Proposition of solutions like the launching of additional communications satellites in the orbit, aggressive deployment of broadband wireless technology, construction of fibre optics backbone around the country to link to Africa-1 hub, Free Space Optics, WiMax Technologies to expand density. The National Communications Commission (NCC) which is the Apex body that allocates spectrums should put up effective control mechanisms for radio frequency users to improve infrastructure and Quality of Service. This paper has also attempted to provide mathematical guide for the accurate estimation of bandwidth requirements for organizations to improve the quality of service provided.

**Keywords:** Nakagami-m distribution, quality of service, parameter estimates

### 1. Introduction

The telecommunications sector has a great deal of importance in Nigeria. This is due to the significant growth that the mobile phone market has experienced over the past few years in Nigeria and its impact on the population. The telecommunications industry that has become key tools for promoting the country's development. Thus, the Nigeria Communication Commission (NCC), the government regulatory agency, sets rates for the services that the telecommunication industries provide. These services and rates include the access charge tariffs and the payments that must be made to mobile phone concessionaires for the use of their networks.

Telecommunication facilities in Nigeria were first established in 1886 by the colonial administration. At independence in 1960, with a population of roughly 40 million people, the country only had about 18,724 phone lines for use. This translated to a teledensity of about 0.5 telephone lines per 1,000 people. The telephone network then consisted of 121 exchanges of which 116 were of the manual (magneto) type and only 5 were automatic. (Okonji, 2013) <sup>[1]</sup> However, the Nigerian telecommunication industry has somewhat toed the path of development of the global telecommunication industry – from state monopoly to liberalization to weak competition to growing competition and to growing service innovation.

As at the country's independence, the telecommunication industry was dominated by the Nigerian Telecommunications Limited (NITEL), a government-owned monopoly operator (Mawoli, 2009) <sup>[2]</sup>. NITEL's services include the provision of Fixed Telephone, Telegraph (gentex), and Payphone and others. Its main objective was to harmonize the coordination of the external and internal telecommunications services, rationalize investments in telecommunications development and provide easy access, efficient and affordable services. Ndukwe (2003) <sup>[3]</sup> noted that between 1987 and 1992, no remarkable improvement was recorded in the performance of NITEL, and consumer demands were largely unmet. This

prompted the Federal Government to embark on market oriented reforms by partially liberalizing telecommunication industry.

This Liberalization actually began in 1993, with the establishment of the Nigerian Communications Commission (NCC) as prescribed by Decree 75 of 1992. However, some segments of the market were still restricted to the monopoly of NITEL (Okonji, 2013) <sup>[1]</sup>. This ceased to be the case in 2001 when NITEL came under the regulatory oversight of NCC, and was formally licensed as an operator. Nigeria's return to democracy in 1999 brought full liberalization of the telecommunication industry, and necessitated the strengthening of the power and independence of the industry regulator, NCC. Consequently, a new telecommunication law was enacted in 2003. This law specifically empowers the NCC to make regulations and guidelines for the industry (Mawoli, 2009) <sup>[2]</sup>.

The period from 2000 till date, could be described as the period of Nigeria's telecommunication revolution, given the enormous growth and innovation registered in the industry. The major auctioning of digital mobile licenses in 2001 spurred many activities in the sector: active subscription grew from 400,000 lines in 2001 to 89.8 million in 2011, resulting in a teledensity of 0.4 and 64.16 per cent in both years respectively (Okonji, 2013) <sup>[1]</sup>.

A modern telecommunications network incorporates a transport network that carries voice and data, and a vital signaling network core that controls and manages voice and data circuits in the transport network. Call setup and other messages that traverse the signaling network are routinely examined by service providers to maintain quality of service, debug network problems and generate billing records. These messages provide a wealth of forensic information about phone calls and calling patterns. Since signaling messages provide data about phone calls, but not the content of phone conversations, and collecting and analyzing these messages may not be subject to the same legal restrictions as recording voice conversations.

This research paper describes techniques for collecting detailed information about phone calls and calling patterns. The techniques can be implemented using current telecommunications network surveillance equipment (Thompson, 2000) [5], and the collected data can be analyzed and stored at little additional cost. There are various statistical models for analyzing communication and telecommunication networks, among which are: probability theory, Markov chains (both discrete and continuous), queuing theory, event-based simulations (confidence intervals and regression models), scheduling theory, and many others.

**2. Research Methodology**

**2.1 Nakagami-m Approach to Fading Network Signal**

In wireless telecommunications, the received signal is subjected to fading due to two physical mechanisms. On one hand, the multipath components cause rapid and deep fading in displacements of few wavelengths (small-scale area). This is the well-known short-term fading or fast fading, which has been extensively analyzed in the literature (Thompson 2000 and Molisch, 2011) [5, 7]. This fast fading signal has been modeled statistically using the Rice, Rayleigh, Nakagami-*m*, and Weibull distributions.

The Nakagami-*m* distribution is frequently employed to model the fast fading since it fits better than the other distributions in many measurement campaigns (Nakagami, 1960 and Rubio, *et al.* 2007) [9]. On the other hand, the received signal fluctuates slowly around a mean in displacements of hundreds of wavelengths (large-scale area). This variation is known as long-term fading or shadowing. This shadowing is due to the temporal blockage of the direct component between the transmitter and receiver terminals. The shadowing is commonly modeled statistically by a lognormal distribution.

Nakagami distribution has been used to model attenuation of wireless signals traversing multiple paths, fading of radio signals, data regarding communicational engineering, and so forth. The distribution may also be employed to model failure times of a variety of products (and electrical components) such as ball bearing, vacuum tubes, and electrical insulation. It is also widely considered in biomedical fields, such as to model the time to the occurrence of tumors and appearance of lung cancer. It has the applications in medical imaging studies to model the ultrasounds especially in Echo (heart efficiency test).

Shanker *et al.* (2005) [10] and Tsui *et al.* (2006) [11] used the Nakagami distribution to model ultrasound data in medical imaging studies. This distribution is extensively used in reliability theory and reliability engineering and to model the constant hazard rate portion because of its memory less property. Yang and Lin (2000) [12] investigated and derived the statistical model of spatial-chromatic distribution of images. Through extensive evaluation of large image databases, they discovered that a two-parameter Nakagami distribution well suits the purpose. Kim and Latchman (2009) [13] used the Nakagami distribution in their analysis of multimedia.

For this research paper, probability theory (Nakagami-*m* distribution) is used in analyzing the telecommunication data.

**2.2 Estimation of Two Parameters of Nakagami-m Distribution**

The probability density function (p.d.f.) of the Nakagami-*m* distribution [1] with two parameters {*m*,  $\Omega$ } is given as:

$$f(x/m, \Omega) = \frac{2m^m}{\Gamma(m)\Omega^m} x^{2m-1} \exp\left(-\frac{m}{\Omega} x^2\right) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m x^{2m-1} \exp\left(-\frac{m}{\Omega} x^2\right) \begin{matrix} 0 \leq x \leq \infty \\ 0.5 < m \leq \infty \\ 0 \leq \Omega \leq \infty \end{matrix} \quad (1)$$

and its cumulative distribution function is given as:

$$F(x/m, \Omega) = \frac{\gamma(m, \frac{m}{\Omega} x^2)}{\Gamma(m)} \quad (2)$$

where *m* is the shape parameter and  $\Omega$  is the speed parameter. Its measures are given as:

$$\text{Mean; } E(x) = s + \left(\frac{\Omega}{m}\right)^{1/2} \frac{\Gamma(m+1/2)}{\Gamma(m)} \quad (3)$$

$$\text{Variance; } Var(x) = \Omega \left[1 - \frac{1}{m} \left(\frac{\Gamma(m+1/2)}{\Gamma(m)}\right)^2\right] \quad (4)$$

$$\text{Skewness; } Skewness = \frac{\Gamma(m+3/2)}{m^{3/2}\Gamma(m)} \quad (5)$$

$$\text{Kurtosis; } Kurtosis = \frac{(m+1)}{m} \quad (6)$$

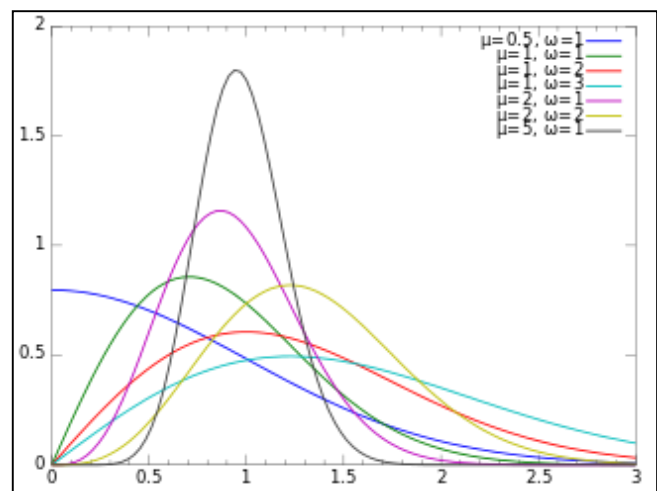
Its parameters are estimated as:

$$m = \frac{E^2(X^2)}{Var(X^2)} \quad (7)$$

And

$$\Omega = E(X^2) \quad (8)$$

The graph below shows the curve of the pdf of the Nakagami-*m* distribution at several values of the two parameters {*m*,  $\Omega$ }:



Source: www.wikipedia.com/probability/Nakagami\_m

**Fig 1:** Nakagami-*m* probability density function curve under different values of *m* and  $\Omega$

**2.3 Estimation of Three Parameters of Nakagami-m Distribution**

The probability density function (p.d.f) of the extended Nakagami distribution with three parameters {*m*,  $\Omega$ , *s*} as defined by Akintunde (2018) [15] and also known as Nakagami-Akintunde distribution is given as:

$$f(x/m, \Omega, s) = \frac{2m^m}{\Gamma(m)\Omega^m} (x-s)^{2m-1} \exp\left(-\frac{m}{\Omega} (x-s)^2\right)$$

$$= \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m (x-s)^{2m-1} \exp\left(-\frac{m}{\Omega}(x-s)^2\right) \begin{matrix} 0 \leq x \leq \infty \\ 0.5 < m \leq \infty \\ 0 \leq \Omega \leq \infty \\ x \leq s \leq \infty \end{matrix} \quad (9)$$

and its cumulative distribution function is given as:

$$F(x/m, \Omega, s) = \frac{\gamma(m, \frac{m}{\Omega}(x-s)^2)}{\Gamma(m)} \quad (10)$$

where  $m$  is the shape parameter,  $\Omega$  is the speed parameter and  $s$  is the location parameter.

The measures of the three parameter distribution are given as:

$$\text{Mean; } E(X) = s + \left(\frac{\Omega}{m}\right)^{1/2} \frac{\Gamma(m+1/2)}{\Gamma(m)} \quad (11)$$

$$\text{Variance; } Var(x) = \Omega \left[ 1 - \frac{1}{m} \left(\frac{\Gamma(m+1/2)}{\Gamma(m)}\right)^2 \right] \quad (12)$$

Skewness;

$$\text{Skewness} = \frac{\left[ s^3 + 3s^2 \left(\frac{\Omega}{m}\right)^{1/2} \frac{\Gamma(m+1/2)}{\Gamma(m)} + 3s\Omega + \left(\frac{\Omega}{m}\right)^{3/2} \frac{\Gamma(m+3/2)}{\Gamma(m)} \right]}{\left[ s^2 + 2s \left(\frac{\Omega}{m}\right)^{1/2} \frac{\Gamma(m+1/2)}{\Gamma(m)} + \Omega \right]^{3/2}} \quad (13)$$

Kurtosis;

$$\text{Kurtosis} = \frac{\left[ s^4 + 4s^2 \left(\frac{\Omega}{m}\right)^{1/2} \frac{\Gamma(m+1/2)}{\Gamma(m)} + 6s^2\Omega + 4s \left(\frac{\Omega}{m}\right)^{3/2} \frac{\Gamma(m+3/2)}{\Gamma(m)} + \frac{\Omega^2(m+1)}{m} \right]}{\left[ s^2 + 2s \left(\frac{\Omega}{m}\right)^{1/2} \frac{\Gamma(m+1/2)}{\Gamma(m)} + \Omega \right]^2} \quad (14)$$

The parameters are estimated as:

$$m = \frac{E^2((X-s)^2)}{Var((X-s)^2)} \quad (15)$$

$$\Omega = E((X-s)^2) \quad (16)$$

And

$$s = \frac{\sum x}{n} \quad (17)$$

The graph below shows the curve of the pdf (9) of the three parameters Nakagami distribution at several values of the three parameters  $\{m, \Omega, s\}$  as shown below:

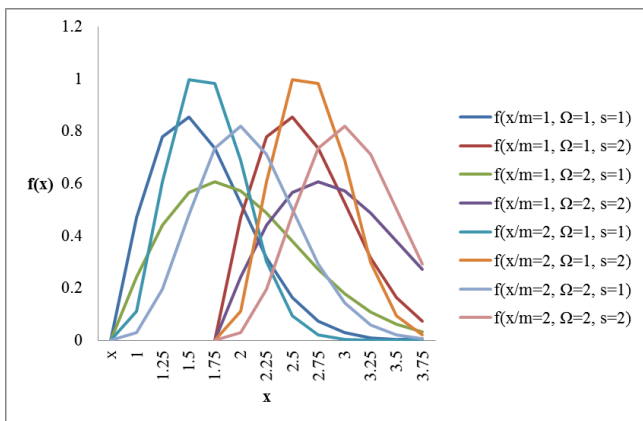
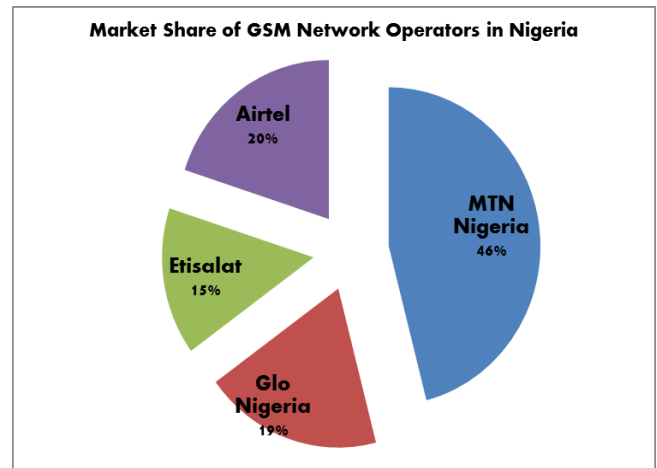


Fig 2: Three Parameters Extended Nakagami probability density function curve under different values of  $m, \Omega$  and  $s$

### 3. Analyses and Results

From available data, a mere 18,724 lines in 1960 has jumped to 110,147,519, while active lines stood at 81,931,223 thus providing total installed capacity of 218,756,182. Studies also show that there is a positive relationship between telecommunication infrastructure development and economic growth (Tella, *et al* 2006). This explains why Nigeria remains the fastest growing market for telecom operators in the world. However, these figures do not suggest that Nigeria has reached her optimum. There are still many remote communities who are yet to smell the usage of telephone nor the Internet. In most cases, these group of citizens have to come to the cities to write and send their mails. This means a lot of work remains to be done in the areas of infrastructure and diffusion. Even at that, there are a lot of bottle-neck problems with the telecommunication services in the urban areas and Nigeria as a whole.

Data from two major Global System of Mobile (GSM) Telecommunication service providers; namely MTN Nigeria and Glo Nigeria are used and analyzed for Call Set-Up Success Rate (CSSR), because of the large market share they enjoy in Nigeria (this is shown in Figure 3 below). Three parameters Nakagami-m distribution is employed.



Source: Nigeria Communication Commission

Fig 3: Market Share of GSM Network Operators in Nigeria

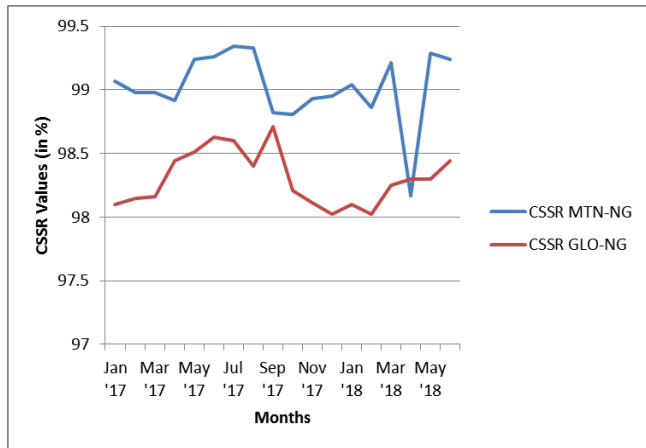
**Definition:** The Call Ste-Up Success Rate (CSSR) is calculated by taking the number of the unblocked call attempts divided by the total number of call attempts. A call set-up is an exchange of signaling information in the call process that leads to traffic channel seizure,

$$CSSR (\%) = \frac{\text{Number of unblocked call attempts}}{\text{Total number of call attempts}} \times 100 \% \quad (18)$$

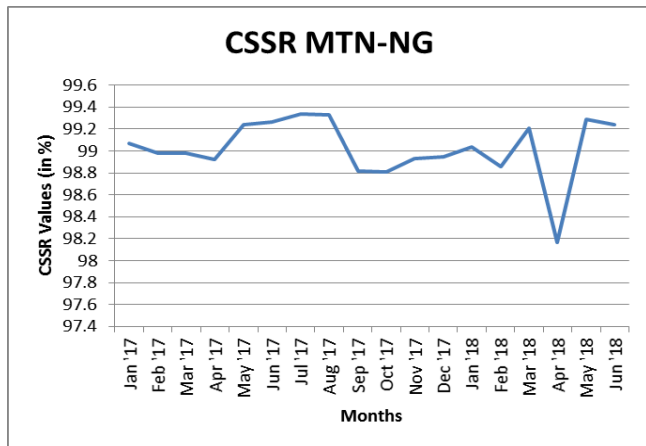
According to Nigerian Communications Commission (NCC), the Call Ste-Up Success Rate (CSSR) performance threshold is set to be 98%. Thus the location parameter,  $s$ , is assumed to be 98. The data analyzed are from January 2017 to June 2018 on a nationwide basis. The data are obtained and analyzed as well estimated and fitted for CSSR using three parameters Nakagami distribution, which are as illustrated by the table and figures below:

**Table 1:** Parameters Estimates of Three-Parameter Nakagami Distribution

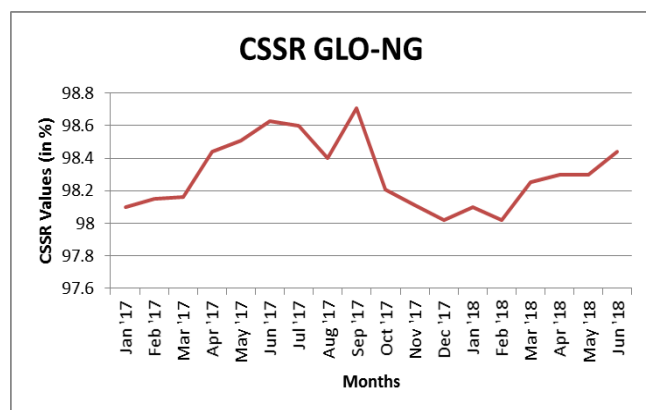
MTN Nigeria	Glo Nigeria
$\Omega = 1.1974$	$\Omega = 0.1624$
$s = 98$	$s = 98$
$m = 1$	$m = 1$



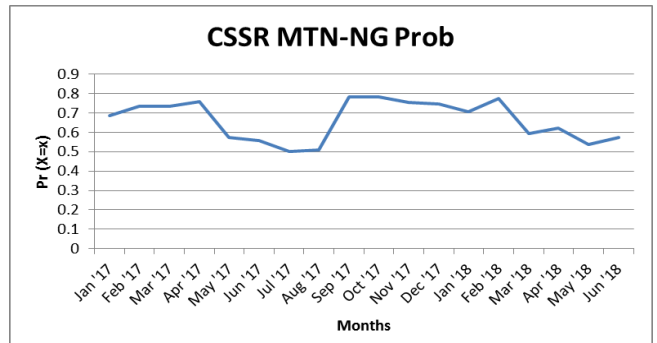
**Fig 4:** CSSR Curve for MTN-NG and GLO-NG



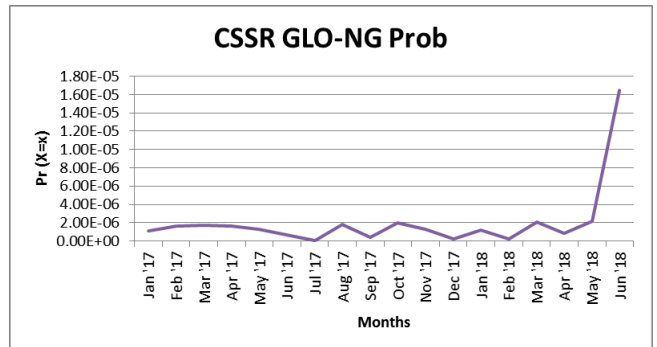
**Fig 5:** CSSR Curve for MTN-NG



**Fig 6:** CSSR Curve for GLO-NG



**Fig 7:** Nakagami Distribution fitted value for CSSR Curve for MTN-NG



**Fig 8:** Nakagami Distribution fitted value for CSSR Curve for GLO-NG

**4. Summary and Conclusion**

It was discovered that the Call Set-Up Success Rate (CSSR) performed better for MTN Nigeria than Glo Nigeria in the telecommunication services provided in Nigeria during the time of study. It was also observed that MTN Nigeria CSSR fitted readily more to CSSR values using Nakagami distribution, while Glo Nigeria CSSR values do not. It is also recommended that the CSSR performance threshold should be increased from 98% by Nigeria Communication Commission to ensure more call success by the users of the telecommunication service providers.

**References**

1. Okonji E. ICT Sector at 53: Tremendous Growth, Poor Services This Day, viewed 2013-2014, <http://www.thisdaylive.com/articles/ict-sector-at-53-tremendous-growth-poor-services/160371/>
2. Mawoli M. Liberalization of the Nigerian Telecommunication Sector: a Critical Review Journal of Research in National Development. 2009; 7(2):102-124.
3. Ndukwe E. An overview of evolution of the telecommunication industry in Nigeria and challenges ahead 1999-2003 Paper presented at the Telecom Summit, viewed 2003-2014, [http://www.ncc.gov.ng/archive/speeches\\_presentations/EVC's%20Presentation/](http://www.ncc.gov.ng/archive/speeches_presentations/EVC's%20Presentation/)

- Overview%20of%20Telecom%20EVC%20Presentatio  
n%20at%20Second%20NTS%20Oct%202003.pdf
4. Nigerian Communications Commission, 2013, Determination of market Dominance in Selected Communication Markets in Nigeria, viewed 13 August 2014, [http://www.ncc.gov.ng/index.php?option=com\\_docman&task=doc\\_download&gid=365&Itemid](http://www.ncc.gov.ng/index.php?option=com_docman&task=doc_download&gid=365&Itemid)
  5. Thompson R. Telephone Switching Systems, Artech, Norwood, Massachusetts, 2000-2000.
  6. Parsons JD. Mobile Radio Propagation Channel, John Wiley & Sons, Chichester, UK, 2nd edition, 2000.
  7. Molisch AF. Wireless Communications, John Wiley & Sons, Chichester, UK, 2nd edition, 2011.
  8. Nakagami M. The m-distribution—a general formula of intensity distribution of rapid fading, in Statistical Methods of Radio Wave Propagation, W. G. Hoffman, Ed., pp. 3–35, Pergamon Press, Oxford, UK, 1960.
  9. Rubio L, Reig J, Cardona N. Evaluation of Nakagami fading behaviour based on measurements in urban scenarios, International Journal of Electronics and Communications. 2007; 61(2):135-138.
  10. Shanker K, Cervantes C, Loza-Tavera H, Avudainayagam S. Chromium toxicity in plants, Environment International. 2005; 31(5):739-753.
  11. Tsui PH, Huang CC, Wang SH. Use of Nakagami distribution and logarithmic compression in ultrasonic tissue characterization, Journal of Medical and Biological Engineering. 2006; 26:2:69-73.
  12. Yang DT, Lin JY. Food availability, entitlement and the Chinese famine of 1959–61, Economic Journal. 2000; 110-460, 136-158.
  13. Kim K, Latchman HA. Statistical traffic modeling of MPEG frame size: experiments and analysis, Journal of Systemics, Cybernetics and Informatics. 2009; 7(6):54-59.
  14. [www.wikipedia.com/probability/Nakagami\\_m](http://www.wikipedia.com/probability/Nakagami_m)
  15. Akintunde Oyetunde A. On The Introduction Of Location Parameter To Nakagami-M Distribution. Published in the International Journal of Statistics and Applied Mathematics. 2018; 3(4):147-154. <http://www.mathsjournal.com/archives/2018/vol3/issue4/PartB>
  16. Sheriffdeen A. Lloyd Ahamefule Amaghionyeodiwe and Bolaji Adesola ADESOYE, 2006, Telecommunications Infrastructure And Economic Growth: Evidence From Nigeria <https://ncc.gov.ng/technology/standards/qos-map>