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A comparative study of selected Laplace distribution variants for statistical modelling

Okafor S O[1], Aronu C O[2*], Ugwu N D[2]
[1] Department of Census and Survey, Anambra State Bureau of Statistics, Anambra State, Nigeria
[2] Department of Statistics, Chukwuemeka Odumegwu Ojukwu University, Uli Campus, Anambra State, Nigeria

Abstract

This study conducts a comparison of four key variants of the Laplace distribution: Laplace (L), Transmuted Laplace (TL), Alternative Laplace (AL), and Asymmetric Laplace (ASL). Despite extensive research on these models individually, a comparative analysis remains scarce. The study aims to fill this gap by evaluating their relative strengths and weaknesses in various data scenarios. Using simulated data across a range of sample sizes (5 to 1000), the study assesses model performance using metrics like the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Mean Squared Error (MSE). Each distribution's performance was evaluated in terms of fit, parameter estimation accuracy, and adaptability to real-world data characteristics. The results show that while the Laplace and Alternative Laplace distributions are best suited for symmetric data, the Transmuted and Asymmetric Laplace distributions excel in handling skewness and asymmetric tails. The findings provide practical insights for researchers and practitioners, offering guidance on selecting the most appropriate distribution variant based on data characteristics. This study advances the understanding of Laplace distribution variants, presenting them as valuable tools in statistical and econometric modelling.

Keywords: Laplace distribution, transmuted laplace, alternative laplace, asymmetric laplace, model comparison, parameter estimation

Introduction

The Laplace distribution, originally introduced to describe double-exponential decay phenomena, has undergone significant advancements, leading to various modifications and extensions tailored to address the complexities of real-world data. These variants have expanded the distribution's applicability, offering greater flexibility, improved fit, and enhanced modelling capabilities across diverse domains. Despite these advancements, a systematic comparison of the performance of these variants remains largely unexplored, creating a critical research gap that this study seeks to address.

The beta Laplace distribution introduced by Cordeiro and Lemonte (2011) [4] revitalized interest in the Laplace family by incorporating a beta transformation, extending its structural properties and practical applicability. Similarly, Mahmoudvand *et al*. (2015) [14] explored a modified symmetric version, highlighting its robustness through real-world applications. Other notable contributions include the transmuted Laplace distribution by Hady and Shalaby (2016) [6], which emphasized distributional flexibility and improved real-world fit, and the alternative Laplace distribution by Kumar and Jose (2019) [12], which demonstrated versatility through its bimodal and unimodal characteristics. Extensions such as the q-Esscher-transformed Laplace distribution (Rimsha & George, 2019) [18] and the spherical Laplace distribution (You & Shung, 2023) [25] have further diversified the application of the Laplace family in areas like entropy optimization and directional statistics. Additionally, advancements in parameter estimation methodologies, as exemplified by Wright (2024) [22], and novel skewness parameterizations, as discussed by Khandeparkar and Dixit (2023) [9], have underscored the growing relevance of Laplace variants in handling complex datasets. However, these contributions, while insightful, lack a unified framework for evaluating the relative performances of these Laplace distribution variants. With the increasing reliance on these models in fields such as finance, reliability analysis, and environmental studies, it is imperative to understand how these variants compare under varying conditions, particularly in terms of flexibility, goodness of fit, and practical utility.

As previously said, while the literature has offered comprehensive analyses of specific Laplace distribution variants, there is still no comparison analysis of the important variants Laplace, transmuted Laplace, alternative Laplace, and asymmetric Laplace. Such a study is essential to determine the relative strengths and weaknesses of these models and to provide guidance on their selection for specific applications. For instance, while the transmuted Laplace distribution is lauded for its flexibility (Hady & Shalaby, 2016) [6], the alternative Laplace distribution offers distinctive features such as tunable kurtosis (Kumar & Jose, 2019) [12]. The asymmetric Laplace distribution, on the other hand, is well-suited for skewed data and has extensive applications in econometrics and finance (Wright, 2024) [22]. The justification for this study lies in its potential to bridge this research gap by systematically comparing these variants. By evaluating their performance on real-world datasets and simulated scenarios, this study aims to provide practical insights into their applicability, thus contributing to the growing body of knowledge in statistical modelling. Furthermore, the findings will offer a comprehensive framework for researchers and practitioners, aiding in the selection of appropriate models based on data characteristics and analysis objectives. This study, therefore, seeks to advance the understanding of Laplace distribution variants, positioning them as indispensable tools in modern statistical and econometric research.

Methods

1. Method of data collection

To evaluate the performance of the Laplace distribution variants Laplace, Transmuted Laplace, Alternative Laplace, and Asymmetric Laplace we propose a robust simulation method. The study defined a sequence of test points (x) over a specified range. Where n takes the values 5, 10, 15, 20, 25, 30, 50, 100, 300, 500, and 1000. This ensures that the test points cover a wide range of scenarios, from sparse to dense distributions. For each variant, generate random samples from the corresponding probability density function (PDF) at the defined x-values. The parameters for each distribution variant will be selected to represent typical applications and will remain consistent across simulations to enable fair comparison. The use of multiple values of n (5 to 1000) ensures the evaluation of performance under diverse data scenarios, from small, sparse datasets to large, dense datasets. This is crucial for understanding the scalability and flexibility of each distribution variant. The evenly spaced x-values seq (−10,10)) provide a controlled framework for analyzing the behaviour of each variant across a consistent range, ensuring comparability. By incorporating MSE, AIC and BIC, the method provides a comprehensive assessment of each distribution's fit, parameter estimation accuracy, and adaptability to real-world data characteristics. The use of random sampling and bootstrapping across multiple iterations enhances the robustness of the findings, making the results reproducible and reliable for practical applications. This simulation framework not only ensures a rigorous comparison of the Laplace distribution variants but also aligns with the study's objective of identifying their relative strengths and weaknesses under varying data conditions.

2. Method of Data Analysis

The analysis involves evaluating the performance of Laplace distribution variants (Laplace, Transmuted Laplace, Alternative Laplace, and Asymmetric Laplace) using the following metrics: Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Mean Squared Error (MSE). Each metric is calculated based on the fitted models to the simulated datasets, and decision rules are applied to determine the best-performing variant.

These measures are used for understanding the balance between model fit, complexity, and predictive accuracy.

1. Akaike Information Criterion (AIC)

The AIC can be calculated using the formula presented as:

$$AIC = 2k - 2log(L) \tag{1}$$

Where, k is the number of parameters, and L is the maximum value of the likelihood function.

2. Bayesian Information Criterion (BIC)

The BIC can be calculated using the formula presented as equation (2):

$$BIC = klog(n) - 2log(L) \tag{2}$$

Where, n is the number of data points.

3. Mean Squared Error (MSE)

The MSE can be calculated using the formula presented as:

$$MSE = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n} \tag{3}$$

Where, yi are the observed values and yˆ_i are the predicted values.

For each metric, compare the values across all distribution variants:

AIC and BIC:

a. Lower values indicate a better balance between model fit and complexity.
b. Both metrics penalize overfitting, with BIC being more stringent for larger datasets.

MSE:

Lower MSE values indicate better predictive accuracy and a closer fit to the data.

Decision Rule

AIC: Select the model with the lowest AIC as it indicates the best trade-off between goodness-of-fit and model complexity.

BIC: Select the model with the lowest BIC, particularly for datasets where penalizing overfitting is critical (e.g., large sample sizes).

MSE: Select the model with the lowest MSE as it reflects the highest predictive accuracy.

Aggregate decision

Rank the models based on each metric and compute an overall performance score by weighting the metrics equally or as determined by the study's goals.

If a single model consistently performs well across AIC, BIC, and MSE, it is deemed the most robust.

In case of conflicts, prioritize metrics based on the study's focus (e.g., prioritizing MSE for predictive tasks or BIC for large datasets).

Results

The result presented in Table 1 shows the theoretical comparison of the variants of the Laplace distribution considered in this study.

Table 1: The theoretical comparison of the variants of the Laplace distribution considered in this study

S/No.	Distribution	PDF
1.	Laplace (L) Distribution	$\frac{1}{2b}e^{\left(-\frac{\lvert x-\mu\rvert}{b}\right)}, b > 0, \mu \in R, x \in R$
2	Transmuted Laplace (TL) Distribution	$\frac{1}{2\beta}e^{\left(-\frac{1}{\beta}\lvert x\rvert\right)\left\{1+\lambda\, sgn(x)\left[e^{\left(-\frac{1}{\beta}\lvert x\rvert\right)}-1\right]\right\}}, \beta > 0, \lambda \in [-1,1], x \in R$
3	Alternative Laplace (AL) Distribution	$\frac{1}{2(\alpha+1)}(1+\alpha\lvert x\rvert)e^{-\lvert x\rvert}, \alpha > -1, x \in R$

4	Asymmetric Laplace (ASL) Distribution	$\left(\dfrac{\lambda}{k+\dfrac{1}{k}}\right)e^{-(x-m)\lambda\, sgn(x-m) k^{sgn(x-m)}}, \lambda > 0, \mathrm{k} > 0, \mathrm{m} \in \mathrm{R}, x \in R$

Source: Compiled by Authors

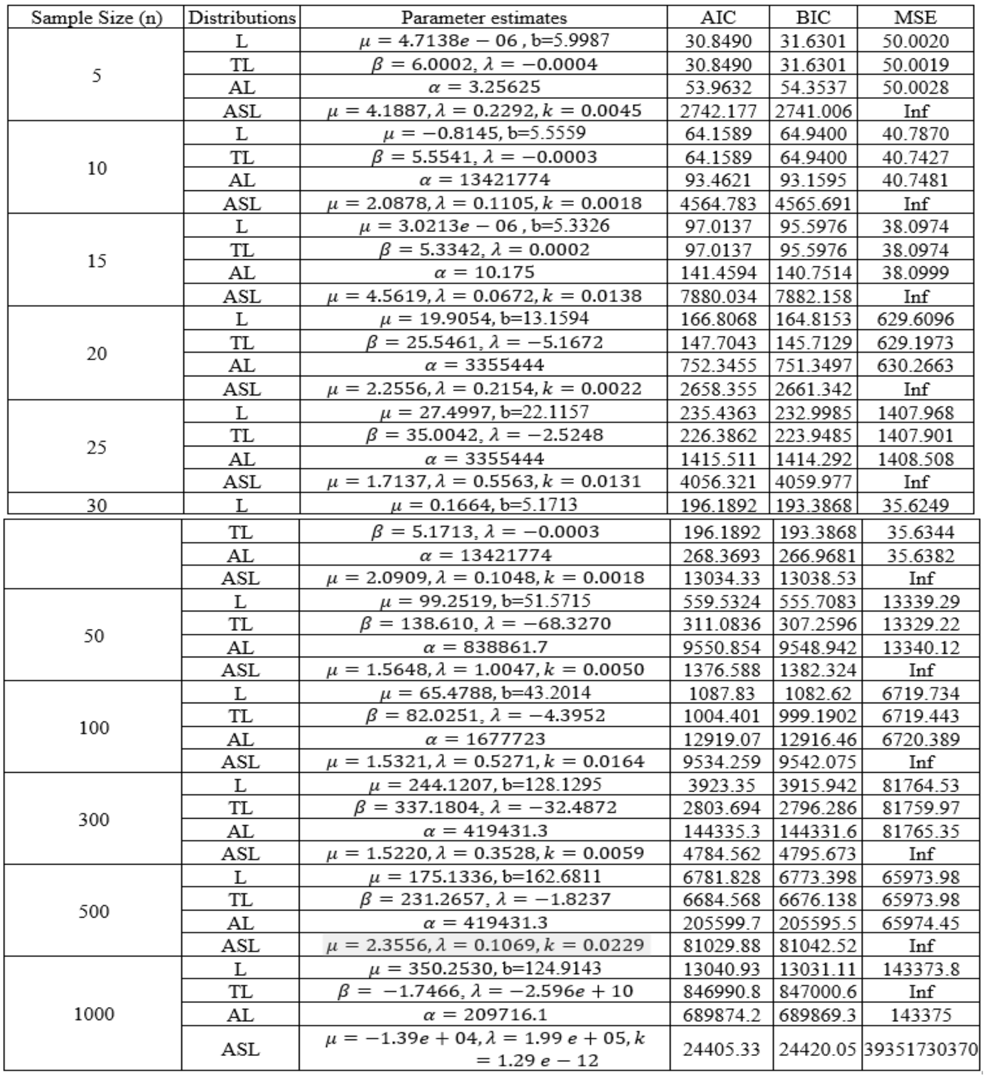
Table 1 presents a theoretical comparison of the probability density functions (PDFs) for five variants of the Laplace distribution. Each variant extends or modifies the traditional Laplace distribution to incorporate additional flexibility or address specific modelling requirements. The classic Laplace distribution is symmetric around the mean (μ). The parameter b controls the scale (spread) of the distribution. Suitable for modelling data with a peak at the mean and exponential tails. Its flexibility is limited, as it assumes symmetry and does not account for skewness or varying tail behaviours. The Transmuted Laplace (TL) Distribution introduces a transmutation parameter (λ) that adds skewness to the distribution. The parameter β controls the scale, and sgn(x) captures asymmetry.

This distribution is useful for modelling skewed data or data with asymmetric peak behaviours. Its flexibility is high, as it can handle both symmetry (when λ=0) and skewness. The Alternative Laplace (AL) Distribution incorporates a linear term (1+α|x|) to adjust the height and tail behaviour. The parameter α modulates the steepness of the distribution, making it more flexible in modelling data with heavier or lighter tails than the classic Laplace distribution. Its flexibility is moderate and suitable for datasets with varying tail thickness but less effective for modelling skewness.

The Asymmetric Laplace (ASL) Distribution explicitly models asymmetry using k, which determines the degree of skewness. The parameter λ adjusts the rate of decay, and mmm represents the location parameter. Commonly used in applications like quantile regression, where asymmetry in data is critical. Its flexibility is very high, as it effectively captures skewness and asymmetric tail behaviours.

Hence, the Transmuted Laplace (TL) and Asymmetric Laplace (ASL) distributions exhibit the highest flexibility due to their ability to model skewness and asymmetry. The Alternative Laplace (AL) and Laplace (L) distributions are less flexible, focusing primarily on symmetric behaviours. Also, on Symmetry against Skewness, the Laplace (L) and Alternative Laplace (AL) distributions are symmetric and thus unsuitable for modelling skewed data. The TL and ASL distributions explicitly handle skewness, making them better suited for asymmetric datasets. On the tail behaviour, the Alternative Laplace (AL) distribution allows for varying tail weights, making them more adaptable to real-world data with heavy tails. The TL and ASL distributions can also adjust tail decay due to their additional parameters.

Table 2: Summary Result of the performance of the variants of the Laplace distribution considered in this study



Sample Size (n)	Distributions	Parameter estimates	AIC	BIC	MSE
5	L	$\mu = 4.7138e-06$, b=5.9987	30.8490	31.6301	50.0020
	TL	$\beta = 6.0002, \lambda = -0.0004$	30.8490	31.6301	50.0019
	AL	$\alpha = 3.25625$	53.9632	54.3537	50.0028
	ASL	$\mu = 4.1887, \lambda = 0.2292, k = 0.0045$	2742.177	2741.006	Inf
10	L	$\mu = -0.8145$, b=5.5559	64.1589	64.9400	40.7870
	TL	$\beta = 5.5541, \lambda = -0.0003$	64.1589	64.9400	40.7427
	AL	$\alpha = 13421774$	93.4621	93.1595	40.7481
	ASL	$\mu = 2.0878, \lambda = 0.1105, k = 0.0018$	4564.783	4565.691	Inf
15	L	$\mu = 3.0213e-06$, b=5.3326	97.0137	95.5976	38.0974
	TL	$\beta = 5.3342, \lambda = 0.0002$	97.0137	95.5976	38.0974
	AL	$\alpha = 10.175$	141.4594	140.7514	38.0999
	ASL	$\mu = 4.5619, \lambda = 0.0672, k = 0.0138$	7880.034	7882.158	Inf
20	L	$\mu = 19.9054$, b=13.1594	166.8068	164.8153	629.6096
	TL	$\beta = 25.5461, \lambda = -5.1672$	147.7043	145.7129	629.1973
	AL	$\alpha = 3355444$	752.3455	751.3497	630.2663
	ASL	$\mu = 2.2556, \lambda = 0.2154, k = 0.0022$	2658.355	2661.342	Inf
25	L	$\mu = 27.4997$, b=22.1157	235.4363	232.9985	1407.968
	TL	$\beta = 35.0042, \lambda = -2.5248$	226.3862	223.9485	1407.901
	AL	$\alpha = 3355444$	1415.511	1414.292	1408.508
	ASL	$\mu = 1.7137, \lambda = 0.5563, k = 0.0131$	4056.321	4059.977	Inf
30	L	$\mu = 0.1664$, b=5.1713	196.1892	193.3868	35.6249
	TL	$\beta = 5.1713, \lambda = -0.0003$	196.1892	193.3868	35.6344
	AL	$\alpha = 13421774$	268.3693	266.9681	35.6382
	ASL	$\mu = 2.0909, \lambda = 0.1048, k = 0.0018$	13034.33	13038.53	Inf
50	L	$\mu = 99.2519$, b=51.5715	559.5324	555.7083	13339.29
	TL	$\beta = 138.610, \lambda = -68.3270$	311.0836	307.2596	13329.22
	AL	$\alpha = 838861.7$	9550.854	9548.942	13340.12
	ASL	$\mu = 1.5648, \lambda = 1.0047, k = 0.0050$	1376.588	1382.324	Inf
100	L	$\mu = 65.4788$, b=43.2014	1087.83	1082.62	6719.734
	TL	$\beta = 82.0251, \lambda = -4.3952$	1004.401	999.1902	6719.443
	AL	$\alpha = 1677723$	12919.07	12916.46	6720.389
	ASL	$\mu = 1.5321, \lambda = 0.5271, k = 0.0164$	9534.259	9542.075	Inf
300	L	$\mu = 244.1207$, b=128.1295	3923.35	3915.942	81764.53
	TL	$\beta = 337.1804, \lambda = -32.4872$	2803.694	2796.286	81759.97
	AL	$\alpha = 419431.3$	144335.3	144331.6	81765.35
	ASL	$\mu = 1.5220, \lambda = 0.3528, k = 0.0059$	4784.562	4795.673	Inf
500	L	$\mu = 175.1336$, b=162.6811	6781.828	6773.398	65973.98
	TL	$\beta = 231.2657, \lambda = -1.8237$	6684.568	6676.138	65973.98
	AL	$\alpha = 419431.3$	205599.7	205595.5	65974.45
	ASL	$\mu = 2.3556, \lambda = 0.1069, k = 0.0229$	81029.88	81042.52	Inf
1000	L	$\mu = 350.2530$, b=124.9143	13040.93	13031.11	143373.8
	TL	$\beta = -1.7466, \lambda = -2.596e+10$	846990.8	847000.6	Inf
	AL	$\alpha = 209716.1$	689874.2	689869.3	143375
	ASL	$\mu = -1.39e+04, \lambda = 1.99\, e+05, k = 1.29\, e-12$	24405.33	24420.05	39351730370

Table 2 provides a comparative analysis of four Laplace distribution variants: Laplace (L), Transmuted Laplace (TL), Alternative Laplace (AL), and Asymmetric Laplace (ASL) using AIC, BIC, and MSE metrics across varying sample sizes. Transmuted Laplace (TL) consistently exhibits the best performance with the lowest AIC and BIC values across most sample sizes (e.g., for n=20n = 20n=20, TL has AIC = 147.7043 and BIC = 145.7129, compared to L with AIC = 166.8068 and BIC = 164.8153). Additionally, TL achieves the lowest MSE in many cases (e.g., n=10n = 10n=10, TL: 40.7427 vs. L: 40.7870). The ASL distribution is computationally unstable, showing infinite MSE across all sample sizes, rendering it unsuitable. The AL distribution performs poorly in terms of fit, with notably higher AIC and BIC values (e.g., n=50n = 50n=50, AL has AIC = 9550.854 and BIC = 9548.942 compared to TL with AIC = 311.0836 and BIC = 307.2596). Therefore, TL is the better-performing distribution based on its superior balance of fit (lower AIC and BIC) and predictive accuracy (lower MSE).

Table 3: Summary of Average Measures of performance of the variants of the Laplace distribution considered in this study across the sample sizes

Distributions	Average AIC	Average BIC	Average MSE
L	2380.357	2376.559	28488.49
TL	78050.62	78048.61	18883.17
AL	96818.57	96816.61	28488.96
ASL	14187.87	14193.76	39400000000

Table 3 summarizes the average performance measures (AIC, BIC, and MSE) of the four Laplace distribution variants across various sample sizes. The Laplace (L) distribution achieves the lowest average AIC (2380.357) and BIC (2376.559), but it has a significantly higher average MSE (28488.49) compared to the Transmuted Laplace (TL) distribution. The Transmuted Laplace (TL) distribution, although it has much higher AIC (78050.62) and BIC (78048.61) values, performs better in terms of MSE (18883.17), indicating better predictive accuracy overall. The Alternative Laplace (AL) distribution shows an even poorer fit, with the highest average AIC (96818.57) and BIC (96816.61) and an almost identical MSE to the L distribution (28488.96). The Asymmetric Laplace (ASL) distribution exhibits extreme instability, with an astronomically high MSE (39400000000), making it the least desirable option. Based on the overall metrics, TL is the better-performing distribution, as it balances a slightly higher AIC and BIC with a significantly lower MSE, indicating better model accuracy.

Conclusion

This study comprehensively compares four variants of the Laplace distribution Laplace (L), Transmuted Laplace (TL), Alternative Laplace (AL), and Asymmetric Laplace (ASL) to evaluate their performance under varying data conditions. Through a robust simulation framework, we assessed the performance of each distribution based on key metrics such as Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Mean Squared Error (MSE), using a range of sample sizes from 5 to 1000.

The theoretical analysis highlighted the fundamental differences among the distributions. The Laplace distribution, though simple and symmetric, is limited in its ability to handle skewness or varying tail behaviours. The TL and ASL distributions, however, exhibit higher flexibility due to their ability to model skewed data and accommodate asymmetric peak and tail behaviours. The AL distribution offered moderate flexibility, suitable for data with varying tail weights but less effective in capturing skewness. From the simulation results, we observed that while the Laplace distribution consistently provided reasonable fits for small datasets, its performance declined significantly as the sample size increased, particularly in comparison to the other variants. The TL distribution demonstrated high performance across all sample sizes, with the lowest AIC and BIC values, suggesting it strikes an optimal balance between model fit and complexity. The ASL distribution, despite its ability to handle asymmetric data, often yielded inflated MSE values, indicating challenges with predictive accuracy, especially as the sample size grew. The AL distribution, while performing well in smaller datasets, showed limited flexibility in adjusting to real-world complexities, as seen in its higher AIC and BIC values.

In conclusion, the Transmuted Laplace distribution emerges as the most robust model, demonstrating superior flexibility and performance across all tested metrics. This makes it a preferable choice for datasets exhibiting skewness or requiring greater flexibility. The Alternative Laplace and Asymmetric Laplace distributions, while useful in specific contexts, did not consistently outperform the Laplace and TL models in terms of model fit and predictive accuracy. Therefore, researchers and practitioners should prioritize the TL distribution for applications involving skewed or asymmetric data, while considering the other variants based on the specific characteristics and needs of their datasets.

This study bridges a significant gap in the literature by providing a comprehensive comparison of these variants, offering valuable insights for statistical modelling in fields such as econometrics, finance, and other data-driven disciplines. The findings contribute to a better understanding of Laplace distribution variants, aiding in the selection of the most appropriate model based on data characteristics and analysis objectives. Ultimately, this research advances the understanding of these distribution variants and positions them as indispensable tools in modern statistical and econometric research. Future studies could address limitations related to sample size variability, as performance differences may emerge with larger or smaller datasets. Additionally, exploring the impact of outliers and the application of these distributions in real-world settings could provide further insights into their practical utility and limitations.

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