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Study of queueing model having single arrival with removable server

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Abstract
This paper represents a queueing models in which server are removed for an exponentially distributed random time when queue is empty. In this paper it is assumed that the server is always available but there are many situations in which the server has to perform the secondary work other than to providing service to the customers which is in queue. For example in bank the clerk has also to maintain their records in addition to serve the customers.

Keywords: poisson arrival, arrival of customers in batches, exponential distribution, queue discipline is FCFS, inverse laplace transformation

Introduction

In most of the work done in the queueing theory it is assumed that the server is always available. Schall & Kleinrock (1992) [3] Pegden and Rosenshine (2002) discussed on vacation queue and derived some results for transient behavior for a M/M/1 Queue. Levy & Yechiali (1993) considered a queueing system in which server is not available at all time. Some studies such as Balachandran, Heyman and Sobel (1991) considered that the server will resume his work when there is at least N customer accumulate and named this as N policies. In this chapter, we consider that server is removed for an exponentially distributed random time when the queue becomes empty and made a policy in which server removed from service for secondary work. The idle time of the server may be utilized. In all these studies arrival and service intensities were taken homogenous. Esien and Tainter (1963) firstly worked with the assumption of the single arrival and service, have obtained steady state solutions of M/M/1 queue, where arrival and service intensities are subject to Poissonian jumps between two states.
Many practical situations can be represented closely by this system.

1. During bad weather period, many a times arrival and departure of the plane breakdown at the airport. In such situations the (time) period, during which arrival and departure rates of the passengers at the custom counters are high and low, can be said to follow each other randomly.
2. The rush hour problem can be taken after completion of a cricket match where all spectators/ customers leave the stadium can be considered as one state and during start of match when arrival of spectators/ customer can be consider ordinary periods as another state.

In this paper we consider queueing models in which server are removed for an exponentially distributed random time when queue is empty. There are many situations in which the server has to perform the secondary work for queue. For example in a bank, the clerk has also to maintain their records in addition to serve the customers.
This paper is divided in to two sections. In section 1 arrival are singly and in section 2 arrivals are in batches of variable size. The joint distribution of number of arrivals and departures up to time t is obtained the result are computed numerically interpreted.

Section-1

This section considered a queueing model where the server may be removed from the service facility for an exponential random time ‘θ’ when there is no customer in the queue.
The following assumptions describe the system:

1. Arrivals arrive under Poisson law with parameter λ.
2. The queue discipline is first come – first serve.
3. The service time distribution is exponential with parameter μ.
4. The various stochastic processes in the system are statistically independent.
5. The server will be removed from its service as soon as it become empty for a negative exponential distribution, with parameter θ.

Notations

$P_{ijR}(t)$	:	Probability that there are exactly i arrivals and j departures at time t and the server is on removed state.
$P_{ijB}(t)$	:	Probability that there are exactly i arrivals and j departures at time t and the server is busy.
$P_{ij}(t)$	:	Probability that there are exactly i arrivals and j departures by time ‘t’, $i \geq j \geq 0$.

Initial condition

$P_{00R}(0) = 1$

$P_{00B}(0) = 0$

The difference-difference-differential equation governing the system are

$$P_{i,i,R}(t) = -\lambda P_{i,i,R}(t) + \mu P_{i,i-1,B}(t) \quad (1)$$

$$P_{i,j,R}(t) = -(\lambda+\theta) P_{i,j,R}(t) + \lambda P_{i-1,jR}(t) \quad (2)$$

$$P_{i,j,B}(t) = -(\lambda+\mu) P_{i,j,B}(t)+\lambda P_{i-1,j,B}(t) + \mu P_{i,j-1,B}(t)+ \theta P_{i,j,R}(t) \quad i > j \geq 0 \quad (3)$$

$$P_{i,j}(t) = P_{i,j,R}(t) + P_{i,j,B}(t)$$

Using Laplace transformation of equation (1) – (3)

$$S\,\overline{P}_{iiR}(s) + \lambda\,\overline{P}_{iiR}(s) = 1 + \mu\,\overline{P}_{ii-1,B}(s)$$

$$\overline{P}_{i,i,R}(s) = \frac{1}{S+\lambda}(1 + \mu\,\overline{P}_{i,i-1,B})$$

$$\overline{P}_{00R}(s) = \frac{1}{S+\lambda} \quad (4)$$

$$S\,\overline{P}_{i,j,B}(s) + (\lambda+\mu)\overline{P}_{i,j,B}(s) = \lambda\,\overline{P}_{i-1,j,B}(s) + \mu\,\overline{P}_{i,j-1,B}(s) + \theta\,\overline{P}_{i,j,R}(s) + \overline{P}_{i,j,B}(0)$$

$$\overline{P}_{i,j,B}(s) = \frac{1}{S+\lambda+\mu}[\lambda\,\overline{P}_{i-1,j,B}(s) + \mu\,\overline{P}_{i,j-1,B}(s) + \theta\,\overline{P}_{i,j,R}(s) + \overline{P}_{i,j,B}(0)] \quad (5)$$

$$\overline{P}_{i,j,B}(s) = \left(\frac{\lambda^i}{(S+\lambda+\mu)^i}\right)\{\lambda^i\,\mu\,\theta\,\overline{\beta}_{1,i,j}^{(\lambda+\mu)(\lambda+\theta)}(s)\}\left(\frac{\lambda^i\mu}{(S+\lambda+\mu)^{i+1}}\right)\overline{P}_{i,j-1,B}(s) \quad (6)$$

Using inverse Laplace transformation of equation (4) - (6).

$$P_{00}(t) = e^{-\lambda t}$$

$$P_{i,0,R}(t) = \lambda^i\,\beta_{1,i}^{(\lambda+\theta)}(t)$$

$$P_{i,j,R}(t) = \lambda^i\,\mu\,\overline{\beta}_{ij}^{\lambda(\lambda+\theta)}(t)\,(\mu\,e^{-\lambda t})^{\delta_{ij}}\,P_{i,j-1,B}(t)$$

$$P_{i,j,B}(t) = \left\{\frac{e^{i(\lambda+\mu)}.t^{i-1}}{\lfloor S-1}\right\}[\lambda^i\mu\,\theta\,\overline{\beta}_{1,1,j}^{(\lambda+\mu)(\lambda+\theta)}(t)]\,.\left(\frac{\lambda^i\mu\,e^{-(\lambda+\mu)}t^i}{\lfloor i}\right)P_{i,j-1,B}(t)$$

$$\sum_{i=0}^{\infty}\sum_{j=0}^{i}\overline{P}_{i,j,R}(s) + \overline{P}_{i,j,B}(s) = \frac{1}{S} \text{ and } \sum_{i=0}^{\infty}\sum_{j=0}^{i}P_{i,j,R}(t) + P_{i,j,B}(t) = 1$$

Hence the verification.

1. Exactly i units arrive in time 't' are

$$\overline{P}_{i0}(s) = \sum_{j=0}^{i}\overline{P}_{i,i,R}(s) + \overline{P}_{i,j,B}(s)\,(1-\delta_{ij})$$

$$P_{i0}(t) = \frac{(\lambda t)^i}{\lfloor i}\,e^{-\lambda t}$$

The mean number of arrivals in time ‘t’ is

$$\sum_{i=0}^{\infty} i\ \overline{P}_{i0}(s) = \sum_{i=0}^{\infty} i \left\{ \frac{\lambda^{i}}{(S+\lambda)^{i+1}} \right\} \quad ; i \geq 0$$

$$= \left\{ \frac{\lambda}{S^{2}} \right\}$$

$$\sum_{i=0}^{\infty} i\ P_{i0}(t) = \{\lambda t\}.$$

The total number of arrivals are not affected by removal period ‘θ’ of the server and the arrivals follow a poison distribution.

2. The mean number of arrivals in time t is

$$\sum_{i=0}^{\infty} i\overline{P}_{i0}(s) = \sum_{i=0}^{\infty} i \left\{ \frac{\lambda^{i}}{(s+\lambda)^{i+1}} \right\} ;i^{3}0 \qquad i > 0$$

$$= \left\{ \frac{\lambda}{s^{2}} \right\}$$

taking laplace inverse of $\sum_{i=0}^{\infty} i\overline{P}_{i0}(s)$ is

$$\sum_{i=0}^{\infty} i\overline{P}_{i0}(t) = (\lambda t)$$

3. The laplace transform $\overline{P}_{0j}(s)$ of the probability $P_{0j}(s)$ that exactly j customers have been served by time t we have

4.

$$\overline{P}_{0j}(s) = \sum_{i=j}^{\infty} \overline{P}_{ijR}(s) + \overline{P}_{ijB}(s)\left(1-\delta_{ij}\right)$$

5. Numerical validity check of inversion of $P_{i,0}$ (t)

Prob. [Exactly i units arrive in time‘t’] = $P_{i,0}$ (t)

λ	a_m	t	I	$\prod_{m=1}^{i} a_m \frac{(\lambda t)^i e^{-\lambda t}}{\lfloor i}$	$P_{i,0}$ (t)
1	0.1	1	1	0.03678	0.03678
	0.2			0.07356	0.7356
	0.3			0.11034	0.11034
2	0.1	2	2	0.014648	0.014648
	0.2			0.02929	0.02929
	0.3			0.043944	0.04394

As batch size increases then Probability of Exactly i units arrive in time ‘t’ is also increases.

Conclusion: we conclude from this that the total number of arrivals are not affected by removal period ‘θ’ of the server and the arrivals follow a poison distribution. Also, we come to result that increase in the batch size tends to increase in the arrival time of the customers.

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